1.1 Place Value of Whole Numbers

**Whole Numbers:** Whole numbers can be listed as 0,1,2,3,4,5,6,7,8,9,10,11,...
The "..." means that the pattern of numbers continues without end.

**Infinity:** Infinity is the concept that numbers continue to get larger and larger with no definite end; after every large number comes even larger numbers. Infinity is that area of numbers beyond and larger than the numbers being dealt with in any given mathematical situation.

**Place Value:**
The particular position of a digit within a number determines its place value. For example, if the digit is 4 places from the left, the number is in the thousands place.

**Period:**
A period is a set of digits separated by a comma. For example, the millions period in the number above is the third period from the right: 438.

When a whole number is written using the digits 0→9 it is said to be in the *standard form*. The position that each digit occupies determines its *place value*.

**Example 1.**
What does the digit 4 mean in each number?

- a. 234,598
- b. 456,901
- c. 24,355,567,222

**Answers:**
- a. 4 thousands
- b. 4 hundred thousands
- c. 4 billions
Your Turn Problem #1
What does the digit 7 mean in each number?

a. 573,289:_________________

b. 213,570_________________

c. 37,218,021,593__________

Writing Numbers in Words

Hyphens: When writing a number between 21 and 99, excluding 30, 40, 50, 60, 70, 80 and 90, a hyphen must be used between the two numbers.
 Examples: 24 → twenty-four  
          73 → seventy-three  
          45 → forty-five (40 is spelled forty, not fourty)

Commas: Commas are used to separate each period. The comma is and must be used only after the words: thousand, million, billion, trillion, etc.
 Examples: three thousand, five hundred eighty-eight  
           five million, twenty-two thousand, eleven

And: The word “and” is only to be used with numbers that have decimals. So for now, do not write the word “and”.

Procedure: When asked to rewrite a number using words:
Proceeding left to right, write in words the number in each period, then write its period name (example: trillion, million, etc...), then write a comma, then proceed to the next period, etc...

Example 2a: Rewrite using words: 72,417
Answer: Seventy-two thousand, four hundred seventeen.
Example 2b. Rewrite using words: 12,503,438,236,473

Answer: Twelve trillion, five hundred three billion, four hundred thirty-eight million, two hundred thirty-six thousand, four hundred seventy-three.

Your Turn Problem #2 Rewrite using words

a) 9,207: ______________________________________

b) 3,429,718: ______________________________________

Writing Words into Numbers

Procedure: When asked to rewrite a written number in words into numbers using digits:

Proceeding left to right, for each period (separated by commas), write the number using digits and place commas where they are placed in the written form using words.

If a period is not written, write “000”, and continue to the next period.

Example 3a: Rewrite using numbers:

Four billion, three hundred million, four hundred eight thousand, forty-six

Answer: 4,300,408,046

Example 3b: Rewrite using numbers: Fifty-six trillion, six million, eight

Answer: 56,000,006,000,008

Your Turn Problem #3

Rewrite using numbers:

a) One hundred thirty-five thousand, four hundred two: ________________

b) Four billion, four hundred eight thousand, forty-six: ________________
1.1 Homework: Place Value of Whole Numbers

Complete the place value system

<table>
<thead>
<tr>
<th>6,</th>
<th>5</th>
<th>3</th>
<th>5,</th>
<th>8</th>
<th>2</th>
<th>0,</th>
<th>7</th>
<th>2</th>
<th>9,</th>
<th>7</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Give the place values for the indicated digits
1. 8 in the numeral 7,831
2. 5 in the numeral 27,573,429
3. 2 in the numeral 208,417,145
4. 4 in the numeral 347,123,831,566
5. 9 in the numeral 429,601
6. 0 in the numeral 8,675,309

Write each of the following in words
7. 21:_______________________________________________________________
8. 941:_______________________________________________________________
9. 3,501:_______________________________________________________________
10. 93,880:_______________________________________________________________
1.1 Homework: Place Value of Whole Numbers cont.

Write each of the following in words

11. 34,058,012: ________________________________________________

12. 7,012,000,030,008: _________________________________________

Write each of the following numbers with digits instead of words


15. Five thousand, eighty-two: _____________

16. One hundred thirty-five thousand, four hundred two: ________________

17. Eight trillion, seven thousand: ______________________________

18. Two million, four hundred thousand, eleven: _____________________

19. Forty-three thousand, nine hundred four: _______________________

20. Eight million, two hundred, eighty: ____________________________
1.2 Rounding and Ordering of Whole Numbers

Accuracy
Accuracy indicates how exact we need to be in expressing a value. For example if you need to know the approximate income of your client last year and she made $42,882.46, you would probably answer “$43,000.” Your accuracy was “to the nearest thousand.”

Rounding Off to a Given Place Value:
Rounding off is the process of changing a number and expressing it to a certain accuracy. We will indicate accuracy by specifying a “given place value”.

**Procedure: When asked to round off to a certain place value:**

1. Underline the digit occupying that place value.
2. Look at the digit to its right.
3. If that digit is 5 or larger, add one to the place value digit.
   - If that digit is less than 5, leave the place value digit alone (do not change).
4. Rewrite all the digits to the right of the place value digit as zeros.

**Example 1.** Round the following to the nearest ten.

a) 68
   1. 68
   2. Since the digit to the right is 5 or larger, add 1 to the tens place.
   **Answer: 70**

b) 492
   1. 492
   2. Since the digit to the right is less than 5, leave the tens value digit alone.
   **Answer: 490**

c) 497
   1. 497
   2. Since the digit to the right is 5 or larger, add 1 to the tens place.
   **Answer: 500**
Your Turn Problem #1
Round the following to the nearest ten.

a) 34 __________

b) 857: __________

c) 6,796: __________

Example 2. Round the following to the nearest hundred.

a) 4,829
   1. 4,829
   2. Since the digit to the right is less than 5, leave the hundred digit alone.
      Answer: 4,800

b) 7,951
   1. 7,951
   2. Since the digit to the right is 5 or larger, add 1 to the hundreds.
      Answer: 8,000

   79 + 1 = 80

      Answer: 100

Example 3. Round the following to the nearest thousand.

a) 76,856
   1. 76,856
   2. Since the digit to the right is more than 5, add 1 to the thousands digit.
      Answer: 77,000

b) 39,476
   1. 39,476
   2. Since the digit to the right is less than 5, leave the thousands digit alone.
      Answer: 39,000
Example 3. Round the following to the nearest thousand.

c) 61,472,887
   1. 61,472,887
   2. Since the digit to the right is more than 5, add 1 to the thousands digit.
   Answer: 61,473,000

Your Turn Problem #3

Round the following to the nearest thousand.

a) 45,378 __________
   b) 425 __________
   c) 789 __________

The process is the same for any place value, thousands, ten thousands, hundred thousands, millions, etc.

Example 4. Round the following to the nearest million.

a) 5,717,364
   1. 5,717,856
   2. Since the digit to the right is more than 5, add 1 to the millions digit.
   Answer: 6,000,000

b) 83,476,455
   1. 83,476,455
   2. Since the digit to the right is less than 5, leave the millions digit alone.
   Answer: 83,000,000

Your Turn Problem #4

Round the following to the nearest ten thousand.

a) 245,211 ________________
   b) 3,442,574 ________________
Comparing Numbers

Whole numbers can be shown on a number line.

0 1 2 3 4 5 6 7 8

From the number line, we can see the order of numbers. For example, we can see that 2 is less than 7 because 2 is to the left of 7. For any two numbers on a number line, the number to the left is always the smaller number, and the number to the right is always the larger number.

We use the inequality symbols < or > to write the order of numbers.

**Inequality Symbols**

For any whole numbers a and b:

1. \( a < b \) (read a is less than b) means a is to the left of b on the number line.
2. \( a > b \) (read a is greater than b) means a is to the right of b on the number line.

\( < \) means greater than

\( > \) means greater than

Note: The inequality symbol must always point toward the smaller number. You could also say it opens up to the larger number.

Examples: \( 16 > 5 \) \( 3 < 8 \)

**Example 5.** Place the correct inequality symbol between the two numbers.

- a) 15 \( \) 12
- b) 0 \( \) 11

**Answers**

- a) 15 > 12
- b) 0 < 11

**Your Turn Problem #5**

Place the correct inequality symbol between the two numbers.

- a) 34 \( \) 17
- b) 25 \( \) 12
1.2 Homework: Rounding and Ordering of Whole Numbers

Round each of the following numbers to the nearest ten

1. 346
2. 3,512

3. 13,515
4. 2,397

5. 4
6. 438

7. 76,429
8. 571,597

9. 111,302

Round each of the following numbers to the nearest hundred

10. 746
11. 3,551

12. 13,961
13. 29,952

14. 712
15. 39,748

16. 711,881
17. 42,973

18. 63
1.2 Homework: Rounding and Ordering of Whole Numbers cont.

Round each of the following to the nearest thousand

19. 7,346 ____________  
20. 17,512 ____________  

21. 19,836 ____________  
22. 459 ____________  

23. 501 ____________  
24. 3,489 ____________  

25. 773,412,521 ____________  
26. 9,805 ____________  

27. 7,000,824 ____________  

Round each of the following to the indicated place value

28. 179,212 : ten thousand  
Answer: ________________  

29. 45,873,712 : hundred thousand  
Answer: ________________  

30. 226,771,333 : million  
Answer: ________________  

31. 341,561,202 : ten million  
Answer: ________________  

Place the correct inequality symbol between the two numbers. (< or >)

32. 15 30  
33. 605 210  
34. 5 10  

35. 24 11  
36. 93 0  
37. 85 134
1.3 Addition of Whole Numbers

Addition of Whole Numbers and Properties under Addition.

If two numbers \( a \) and \( b \) are added, that operation can be expressed as \( a + b = c \) where \( a \) and \( b \) are called *addends* and the result \( c \) is called the *sum*.

Addition is the operation where one amount is combined with another amount to get a total.

Example:

\[
\begin{array}{c}
5 \\
+ 2 \\
\hline
7
\end{array}
\]

Phrases for Addition

- *sum*
- *total*
- *increased by*
- *more than*
- *added to*

Example: The sum of 5 and 9.

Translates to: \( 5 + 9 = 14 \)

Example: 4 more than 10

Translates to: \( 4 + 10 = 14 \)

\[ \text{or } 10 + 4 = 14 \]

Notice that the order for addition does not matter. \( 4 + 10 \) is the same as \( 10 + 4 \).

This is called the Commutative Property for Addition.

**Commutative Property for Addition**

\[ a + b = b + a \] where \( a \) and \( b \) are numbers.

Example: \( 5 + 7 = 7 + 5 \)

\[ 12 = 12 \]
**Additive Identity**

The number 0 is the additive identity because for any number a,

\[ a + 0 = a \quad \text{and} \quad 0 + a = a. \]

The number 0 is the identity element of addition because when 0 is added to any number, the resulting sum of addition is the original number.

Example: \( 5 + 0 = 5 \)

---

**Parentheses**

Parentheses: a mathematical expression which indicates: do inside first.

\[
(\quad )
\]

Example: \( 5 + (7 + 3) \)

\[
5 + 10 \quad \text{Since there are parentheses, we will do that first.}
6
15
\]

Note: We would get the same answer if we added the 5 and 7 first. The order does not matter because the order in which the addition is performed will not matter.

Since the order does not matter; i.e., \( 5 + (7 + 3) = (5 + 7) + 3 \) (both equal 15), this gives another property.

This is called the Associative Property for Addition.

---

**Associative Property for Addition**

\[ a + (b + c) = (a + b) + c \] where a, b and c are numbers.

Example: \( 4 + (9 + 5) = (4 + 9) + 5 \) The numbers are in the same order. The parentheses are in different positions.
Your Turn Problem #1
State the indicated Property:

a) 12 + 0 = 12

b) 3 + 7 = 7 + 3

c) 3 + (5 + 8) = (3 + 5) + 8

Procedure: To solve an addition problem where the addends are written horizontally:
1. Rewrite the problem aligning digits by place value - - one’s place on right, ten’s place second from right, etc...
2. Then add the columns, carrying as you go.

Example: Add: 8456 + 2484
Rewrite the problem vertically
Add ones. We get 10 ones. Write the 0 in the ones column and 1 above the tens. This is called carrying.
Add tens. We get 14 tens. Write 4 in the tens column and 1 above the hundred. Add hundreds. We get 9 hundreds. Write 9 in the hundreds column. Add thousands. We get 10 thousands

Your Turn Problem #2

Find the sum of 2,967 and 12,844.

Answer: ____________

Key words such as “total” and “increased by,” will imply addition.
Example 3. The yearly profit for DR Construction was $78,216 in 2004, $153,917 in 2005, and $85,098 in 2006. What is the total profit for these three years?

Solution: The key word in this problem is “total”.

\[
\begin{array}{c}
78216 \\
153917 \\
+85098 \\
\hline
317231 \\
\end{array}
\]

The total profit for the three years was $317,231.

Your Turn Problem #3
Laura’s yearly salary is $58,500. Next year, her salary will increase by $7,000. What will her yearly salary be next year?

Answer: ___________
1.3 Homework: Addition of Whole Numbers

Name the property that is illustrated

1. \(5 + 6 = 6 + 5\) : ___________________________________________________________________

2. \(3 + (9 + 12) = (3 + 9) + 12\) : ___________________________________________________________________

3. \(4 + (5 + 6) = (4 + 5) + 6\) : ___________________________________________________________________

4. \(7 + 0 = 7\) : ___________________________________________________________________

5. \((11 + 4) + 5 = 11 + (4 + 5)\) : ___________________________________________________________________

6. \(11 + 3 = 3 + 11\) : ___________________________________________________________________

Add the following

7. \(12 + 5\) 8. \(17 + 9\)

9. \(23 + 16\) 10. \(33 + 27\)

11. \(78 + 35\) 12. \(96 + 47\)

13. \(30 + 49\) 14. \(93 + 89\)

15. \(321 + 156\) 16. \(872 + 731\)

17. \(507 + 299\) 18. \(857 + 765\)

19. \(941 + 873\) 20. \(3,501 + 2,916\)
1.3 Homework: Addition of Whole Numbers cont.

21. $93,880 + 45,371$

22. $1,342 + 978 + 128$

23. $85,864 + 72,586$

24. $542,682 + 97,388$

25. $727 + 4,871 + 609$

26. $855 + 49,444 + 7,260$

27. Find the total of 23 and 17.

28. Find the sum of 867 and 5309

29. Find a number that is 18 more than 154.

30. What is the sum of 316 and 400?

31. A bowler scored 201, 157, and 198 in three games. What was the total score for those games?

32. Driving to California, Allison drove 871 miles on Monday, 612 miles on Tuesday, and 977 miles on Wednesday. How many miles did she drive on the three days?

33. Corrine spent $364 for tuition, $583 for books, and $35 for parking during one semester. What was the total cost for tuition, books, and parking for that semester?
1.4 Subtraction of Whole Numbers

**Subtraction** can be expressed by the equation \( a - b = c \), where \( a \) is the *minuend*, \( b \) is the *subtrahend*, and \( c \) is the *difference*. \( a - b = c \) is only true if the inverse \( c + b = a \) is true.

Subtraction is the operation where one amount is "taken away" from another amount leaving the difference.

Example: 

\[
\begin{array}{c}
18 \quad \text{minuend} \\
- 5 \quad \text{subtrahend} \\
13 \quad \text{difference}
\end{array}
\]

**Phrases that indicate Subtraction:**

- Minus
- Subtract
- Takeaway
- Difference
- *Subtracted from*
- *Less than*
- Decreased by

**Example 1:**

<table>
<thead>
<tr>
<th>Expressed</th>
<th>Translated</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 minus 4</td>
<td>12 − 4</td>
</tr>
<tr>
<td>The difference of 9 and 2</td>
<td>9 − 2</td>
</tr>
<tr>
<td>5 subtracted from 17</td>
<td>17 − 5 *</td>
</tr>
<tr>
<td>4 less than 10</td>
<td>10 − 4 *</td>
</tr>
</tbody>
</table>

**Note:** The order for the phrases with * are reversed

In this section, we are only working with whole numbers, i.e., 0, 1, 2, 3 … We are not working with negative numbers, but we understand a little about them. If the temperature is \(-17^\circ\), it’s cold. If your checking account is $–24, you’re in the negative.

So, 4 less than 10 translates to \(10 − 4\). If you did it incorrectly, and wrote \(4 − 10\), the answer would be \(-6\). Subtraction is not commutative; order does matter.
Your Turn Problem #1
Translate and Simplify

a) 15 subtracted from 49.

b) The difference of 8 and 5.

Answer:_______  Answer:_______

Borrowing

Borrowing is the process that is used when the lower digit is larger than the upper digit.

Procedure for Subtraction - Borrowing:

1. Line out the digit to the left of the upper digit, subtract 1 from it and write this new digit above.
2. Line out the original upper digit, add 10 to it, and write this number above.
3. Subtract the lower digit.
4. Proceed to the next column on the left.

Example 2a:

\[
\begin{array}{c}
572 \\
- 447
\end{array}
\]

Solution:

\[
\begin{array}{c}
6 \\ \underline{\phantom{1}} 12 \\
5 \quad 4 \\
\underline{\phantom{1}} 7 \\
1 \quad 2 \quad 5
\end{array}
\]

Answer: 125

Example 2b:

\[
\begin{array}{c}
853 \\
- 368
\end{array}
\]

Solution:

\[
\begin{array}{c}
7 \\ \underline{\phantom{1}} 14 \\
8 \quad 13 \\
\underline{\phantom{1}} 8 \\
4 \quad 8 \quad 5
\end{array}
\]

Answer: 485
**Borrowing from 0.**

When the digit(s) to the left of the upper digit is 0, the 0(s) are lined out and replaced by 9(s), and then the first non zero digit to the left is borrowed from.

**Example 2c:**

\[
\begin{array}{c}
4002 \\
\underline{-2375}
\end{array}
\]

Since 5 is greater than 2, borrow a “1” from the 4 and write 9’s in front of the zeros up to the last number. The last number gets a 1 in front of it.

\[
\begin{array}{c}
399 \\
0012 \\
\underline{-2375}
\end{array}
\]

\[= 1627\]

**Your Turn Problem #2**

Subtract

- a) \(786 - 528\)
- b) \(45,354 - 29,676\)
- c) \(98,000 - 17,827\)

a) Answer:_______  b) Answer:_______  c) Answer:_______

**Example 3:** In June the Big Bear Boutique sold $24,760 worth of merchandise, but in July, it sold only $19,458 worth of merchandise. How much more did the boutique sell in June than in July?

**Solution:** “How much more” indicates subtraction.

\[
\begin{align*}
24,760 & \quad - 19,458 \\
& = 5,302
\end{align*}
\]

**Answer:** The boutique sold $5,302 more in June than July.

**Your Turn Problem #3**

The attendance for a concert was 12,329 on Friday and 23,421 on Saturday. How many more people attended on Saturday than on Friday?

Answer: ____________ more people attended on Saturday than on Friday.
1.4 Homework: Subtraction of Whole Numbers

Perform the indicated subtraction

1. \(25 - 12\) 
2. \(95 - 22\) 

3. \(73 - 45\) 
4. \(82 - 24\) 

5. \(34 - 28\) 
6. \(77 - 39\) 

7. \(43 - 29\) 
8. \(38 - 19\) 

9. \(63 - 28\) 
10. \(142 - 98\) 

11. \(781 - 362\) 
12. \(543 - 326\) 

13. \(80 - 34\) 
14. \(70 - 42\) 

15. \(100 - 73\) 
16. \(400 - 156\) 

17. \(900 - 713\) 
18. \(500 - 242\) 

19. \(900 - 537\) 
20. \(600 - 492\) 

21. \(4000 - 2254\) 
22. \(8000 - 5317\) 

23. \(12000 - 6337\) 
24. \(14000 - 8773\) 

25. \(346 - 279\) 
26. \(3,512 - 2,863\)
1.4 Homework: Subtraction of Whole Numbers cont.

27. 13,521 − 5,931

28. 2,000 − 741

29. 15,000 − 9,215

30. 970 − 548

31. 1,208 − 653

32. 6,200 − 4,737

33. 90,000 − 15,605

34. 16 − 5 − 8

35. 34 − 15 − 9

36. 78 − 23 − 17

37. 512 − 73 − 87

38. 900 − 156 − 98

39. 834 − 93 − 185

40. Subtract 23 from 47

41. Subtract 965 from 14,811

42. Find the difference of 24 and 17

43. Find the difference of 76 and 24

44. David's monthly pay of $2600 was decreased by $475 for withholding. What amount of pay did he receive?

45. Henry has $728 in cash and wants to buy a laptop that costs $1,241. How much more money does he need?

46. Brian’s checking account has a balance of $575. Brian wrote three checks for $54, $37, and $143. What was the new balance in the checking account?
1.5 Multiplication of Whole Numbers

Multiplication is the process that shortens repeated addition with the same number. For example, \(8 + 8 + 8 + 8 + 8 = 5 \times 8 = 40\)  (Multiplication symbol: \(\times\) or \(\cdot\))

The multiplication of two numbers \(a\) and \(b\) can be expressed by the equation \(a \cdot b = c\) where \(a\) and \(b\) are factors and \(c\) is called the product (\(a\) and \(b\) are also called multiplicands).

Example: \[4 \times 3 = 12\]

Notice that the order for multiplication does not matter. \(4 \times 10\) is the same as \(10 \times 4\). This is called the Commutative Property of Multiplication.

**Commutative Property of Multiplication**

\[a \times b = b \times a\] where \(a\) and \(b\) are numbers.

**Example:** \(5 \times 7 = 7 \times 5,\) \(35 = 35\)

**Multiplicative Identity**

The number 1 is the identity of multiplication because for any number \(a\), \(a \times 1 = a\) and \(1 \times a = a\).

The number 1 is the identity element of multiplication because when 1 is multiplied by any number, the resulting product is the original number. **Example:** \(5 \times 1 = 5\)

**Associative Property of Multiplication**

\[a \cdot (b \cdot c) = (a \cdot b) \cdot c\] where \(a\), \(b\) and \(c\) are numbers.

**Example:** \((9 \cdot 5) \cdot 7 = 9 \cdot (5 \cdot 7)\)

The numbers are in the same order. The parentheses are in different positions.
**Multiplication Property of Zero**
Multiplying any whole number by 0 gives the product 0.

Example: $5 \cdot 0 = 0$

**Distributive Property of Multiplication over addition**
For any whole numbers $a$, $b$, and $c$, $a(b + c) = a \cdot b + a \cdot c$

Example: $5(7 + 3) = 5 \cdot 7 + 5 \cdot 3$ (verify the result is 50 on both sides.)

**Your Turn Problem #1:**
State the indicated Property: Answers

<table>
<thead>
<tr>
<th>Property</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) $12 \cdot 1 = 12$</td>
<td>a) ______________________</td>
</tr>
<tr>
<td>b) $3 \cdot 7 = 7 \cdot 3$</td>
<td>b) ______________________</td>
</tr>
<tr>
<td>c) $3(5 + 2) = 3 \cdot 5 + 3 \cdot 2$</td>
<td>c) ______________________</td>
</tr>
<tr>
<td>d) $4 \cdot 0 = 0$</td>
<td>d) ______________________</td>
</tr>
<tr>
<td>e) $4 \cdot (5 \cdot 3) = (4 \cdot 5) \cdot 3$</td>
<td>e) ______________________</td>
</tr>
</tbody>
</table>

**Multiplication Facts**
Even though we live in a society where calculators are cheap and readily available, to be competent and confident in mathematics, you should have memorized the multiplication facts for one digit numbers.
### Multiplication Table

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
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### Multiplication of Multiple Digit Factors

**Procedure:** If presented with a problem where one or more of the factors have more than one digit

1. **Beginning with the one’s place digit of the factor written below,** multiply by all the digits of the factor written above (carrying as you go) writing the product directly below (called the partial product).

2. **Next with the ten’s place digit of the factor below,** multiply it by all the digits written above (carrying above). Place a zero to the right and then the partial product from Step 1.

3. **Continue this process until all the digits in the factor below have been multiplied.**

4. **Add the partial products for the product to the problem.**

**Example:**

\[
\begin{align*}
27 & \times 14 \\
\times 284 & \\
108 & \quad 2248 \\
270 & \quad 44960 \\
378 & \quad 112400 \\
& \quad 159608
\end{align*}
\]
Your Turn Problem #2
Multiply the following.

a) \(24 \times 9\)  

b) \(75 \times 38\)

Answer:_________  Answer:_________

Multiplying with three factors.

Procedure: If presented a problem where there factors are written horizontally:

1. Rewrite the first two and find their product.
2. Second, multiply their product by the third factor. Continue this process until all the factors have been multiplied.

Example: Multiply: \(24 \times 8 \times 17\).

Solution:
First, find the product of 24 and 8  
\[
\begin{array}{c}
24 \\
\times 8 \\
\hline \\
192 \\
\end{array}
\]

2\(^{\text{nd}}\), multiply this result with the next number 17
\[
\begin{array}{c}
192 \\
\times 17 \\
\hline \\
1344 \\
\end{array}
\]
\[
\begin{array}{c}
1344 \\
\end{array}
\]
\[
\begin{array}{c}
+1920 \\
\hline \\
3264 \\
\end{array}
\]

Answer: \(3264\)

Your Turn Problem #3
Multiply the following.

a) \(12 \times 7 \times 5\)  

b) \(24 \times 15 \times 37\)

Answer:_________  Answer:_________
**Multiplying by multiples of 10.**

**Example:** Multiply: \(40 \times 30\)

**Solution:**

\[
\begin{array}{c}
40 \\
\times 30 \\
\hline
00 \\
+1200 \\
\hline
\end{array}
\]

Answer: 1200

Notice we can take a shortcut by multiplying the 4 and 3 and then writing the same number of zeros at the end of each number being multiplied.

**Example:** \(600 \times 40\).  

**Solution:** \(6 \times 4 = 24\), and there are three zeros at the end.  

Answer: 24,000

**Your Turn Problem #4**  
Multiply the following.  

a) \(900 \times 70\)  
b) \(3500 \times 400\)  

Answer:_________  
Answer:_________
1.5 Homework: Multiplication of Whole Numbers

Name the property that is being illustrated. (Do not evaluate, just name the property)

1. \(5 \cdot 8 = 8 \cdot 5:\) ________________________________

2. \(8 \cdot 1 = 8:\) ________________________________

3. \(5 \cdot 0 = 0:\) ________________________________

4. \(7 \cdot 6 = 6 \cdot 7:\) ________________________________

5. \(8 \cdot (3 \cdot 5) = (8 \cdot 3) \cdot 5:\) ________________________________

6. \(0 \cdot 8 = 0:\) ________________________________

7. \(2(4 + 9) = (2 \cdot 4) + (2 \cdot 9):\) ________________________________

8. \(1 \cdot 5 = 5:\) ________________________________

9. \(4 \cdot (3 \cdot 5) = (4 \cdot 3) \cdot 5:\) ________________________________

Perform the multiplication.

10. \(12 \times 8\)  

11. \(15 \times 7\)  

12. \(24 \times 8\)  

13. \(36 \times 7\)  

14. \(67 \times 23\)  

15. \(68 \times 36\)  

16. \(95 \times 64\)  

17. \(48 \times 39\)
1.5 Homework: Multiplication of Whole Numbers cont.

18. $46 \times 38$  
19. $62 \times 11$

20. $104 \times 17$  
21. $346 \times 24$

22. $652 \times 18$  
23. $521 \times 78$

24. $234 \times 112$  
25. $206 \times 108$

26. $612 \times 430$  
27. $450 \times 210$

28. $616 \times 911$  
29. $592 \times 300$

30. $6218 \times 98$  
31. $3217 \times 445$

32. $4121 \times 342$  
33. $13 \times 16 \times 5$

34. $127 \times 34 \times 62$  
35. $128 \times 216 \times 52$

36. $5000 \times 300$  
37. $3600 \times 200$

38. $7000 \times 40$  
39. $12000 \times 700$
1.5 Homework: Multiplication of Whole Numbers cont.

40. Find the product of 12 and 7

41. Find the product of 42 and 512

42. One tablespoon of olive oil contains 125 calories. How many calories are in 3 tablespoons of olive oil?

43. The textbook for a course in history costs $54. There are 35 students in the class. Find the total cost of the history books for the class.

44. The seats in the mathematics lecture hall are arranged in 12 rows with 34 seats in each row. Find how many seats are in this room.

45. An apartment building has three floors. Each floor has five rows of apartments with four apartments in each row. How many apartments are in the building?

46. A Koi farm has 7 fish ponds. If each pond has 2000 Koi, how many Koi does the farm have?

47. Each classroom in the Technology Center has 36 computer. If the Technology Center has 12 classrooms, how many computers does the Technology Center have in its classrooms?

48. A year has 365 days. How many days are there in 18 years?
1.6 Division of Whole Numbers

Division defines the operation of finding how many groups of a certain number (the divisor) are contained in another number or amount (the dividend). The answer is the quotient.

Terminology

\[ \frac{a}{b} \rightarrow c \]

This is equivalent to \( b \div a = c \)

The following are equivalent forms of division.

\[ 12 \div 3, \quad 3 \longdiv{12} \]

Both of these division problems equal 4. (12 is the dividend, 3 is the divisor, and 4 is the quotient.)

Related Multiplication Sentence

The division problem of \( 20 \div 5 \) is defined to be the number that when multiplied by 5 gives 20.

\[ 20 \div 5 = 4 \quad \text{related multiplication sentence: } 5 \times 4 = 20 \]

(The last two numbers, 5 and 4, multiply to equal the first number, 20.)

Write the related multiplication sentence to:

\[ 63 \div 9 = 7 \]

\[ \square \times \square = \square \]

Division by 1

Any number divided by 1 is that same number:

\[ a \div 1 = a, \quad \frac{a}{1} = a \]

Example: \( 12 \div 1 = 12, \quad \frac{12}{1} = 12 \)
Dividing a number by itself

Any number divided by itself is 1.

\[ a \div a = 1 \ , \ a \div a \]

Example: \( 12 \div 12 = 1 \ , \ 12 \div 12 \)

Your Turn Problem #1

Divide the following

a) \( 21 \div 21 \)  

Answer: ________  

b) \( 34 \div 1 \)  

Answer: ________  

Dividing into zero (zero is the dividend)

Zero divided by any nonzero number is zero.

\[ 0 \div a = 0 \ , \ a \div 0 \]

Example: \( 0 \div 12 = 0 \ , \ 0 \div 12 \)

(The last two numbers, 12 and 0, multiply to equal the first number, 0.)

Division by zero (zero is the divisor)

A nonzero number divided zero is undefined (Not Zero!).

\[ a \div 0 = \text{undefined} \ , \ 0 \div 12 : \text{undefined} \]

Example: \( 12 \div 0 = \text{undefined} \ , \ 0 \div 12 : \text{undefined} \)

\( 12 \div 0 = 0 \) would be incorrect.

(The last two numbers, 0 and 0, do not multiply to equal the first number, 12.)
Zero divided by zero
When zero is divided by zero, it is called indeterminate.

\[ 0 \div 0 = \text{indeterminate}, \quad 0 \div 0 = \text{indeterminate} \]

The reason \( 0 \div 0 \) is indeterminate is because we can actually write any number and the related multiplication sentence would be true.

**Example:** Choose any number to write after the “=” sign. \( 0 \div 0 = 12 \)

The related multiplication sentence would be \( 0 \times 12 = 0 \) which is true.
So we could write any number then and the related multiplication statement would be true.

Please note: Some students have a tendency to draw a line through zeros, \( \varnothing \). Well this symbol has a meaning to it. It is “no solution”. Please do not use this symbol right now. It is used to say that there is no solution to an equation. We are not solving equations right now. So if the answer is undefined, you must write undefined. If the answer is indeterminate, you must write indeterminate. If the answer is zero, you must write 0.

<table>
<thead>
<tr>
<th>Your Turn Problem #2</th>
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<tbody>
<tr>
<td>Divide the following</td>
</tr>
<tr>
<td>a) ( 43 \div 0 )</td>
</tr>
</tbody>
</table>

| a) ___________ | b) ___________ | c) ___________ |

**Long Division Procedure:**

1. Write the problem in long division format.
   
   If problem is given in this form "\( \text{dividend} \div \text{divisor} \)", change to: \( \text{divisor} \overline{\text{dividend}} \).

   \[ 4,224 \div 12 \quad \text{change to} \quad 12 \overline{4224} \]
2. Determine the placement of the first digit on left in the quotient by determining how many digits of the dividend are necessary to accommodate the divisor.

Take the first digit on the left in the dividend "4".
Now ask, will \(12 \div 4\)? Answer: No.

Now take the first two digits on left in the dividend "42". Ask, will \(12 \div 42\) ?
Answer: Yes

Therefore, the first digit on the left in the quotient is directly above the last digit of 42.

\[
\begin{array}{c}
\downarrow \\
12 \overline{) 4224} \\
\end{array}
\]

3. Determine how many groups of the divisor are contained in the number obtained in Step 2.

How many 12’s are contained in 42? We want to find “what number multiplied by 12 will give us a result as close to 42 without going over”.
\(12 \times 4 = 48\) (too big) \(12 \times 3 = 36\)
Answer is 3.

\[
\begin{array}{c}
3 \\
\downarrow \\
12 \overline{) 4224} \\
\end{array}
\]

4. Multiply the digit obtained from the quotient in step 3 by the divisor. Place that product below the dividend with the digit on the right directly below the digit in the quotient.
\(12 \times 3 = 36\)

\[
\begin{array}{c}
3 \\
\downarrow \\
12 \overline{) 4224} \\
36 \\
\end{array}
\]

5. Subtract product from dividend and then bring down next digit from dividend on right (use an arrow)

\[
\begin{array}{c}
3 \\
\downarrow \\
12 \overline{) 4224} \\
-36 \downarrow \\
\hline \\
62 \\
\end{array}
\]
6. Go back to Step 3, and repeat the process using the bottom line as the new dividend.

How many "12's in 62? 
Answer is 5; 12 × 5 = 60.
Place 5 in the quotient, then continue steps 4 and 5.

\[
\begin{array}{c|c}
35 & 4224 \\
12 & -36 \\
\hline & 62 \\
\hline & -60 \\
\hline & 24 \\
\end{array}
\]

7. Repeat the process using the bottom line as the new dividend.

How many "12's in 24?
Answer is 2; 12 × 2 = 24.
Place 2 in the quotient, then continue steps 4 and 5.

\[
\begin{array}{c|c}
352 & 4224 \\
12 & -36 \\
\hline & 62 \\
\hline & -60 \\
\hline & 24 \\
\hline & -24 \\
\hline & 0 \\
\end{array}
\]

Answer: 352

You can check by multiplying the divisor and the quotient. This will give the dividend.
12 × 352 = 4224

Your Turn Problem #3
Divide the following

a) 6384 ÷ 12 

b) 8551 ÷ 17

\[
\begin{array}{c}
a) \quad \underline{\quad} \\
b) \quad \underline{\quad} \\
\end{array}
\]
Division with Remainders
For this section, when a division problem does not work out evenly, write the remainder preceded by an "R". Later we will do division with fractions and decimals. **You can check by multiplying the divisor and the quotient and adding the remainder**. This result should equal the dividend.

Example: \(361,399 \div 86\).

Solution:

\[
\begin{array}{c|c}
86 & 361,399 \\
\hline
4 & 202 \\
\hline
344 & \\
344 & \\
\hline
173 & \\
172 & \\
\hline
19 & \\
19 & \\
\hline
0 & \\
199 & \\
172 & \\
\hline
27 & \\
27 & \\
\hline
\end{array}
\]

Answer: \(4,202 \, R \, 27\)

Check

\[
\begin{align*}
4,202 \\
\times 86 \\
\hline
361,372 \\
+ 27 \\
\hline
361,399
\end{align*}
\]

It checks.

Your Turn Problem #4
Divide the following

a. \(498 \div 7\)  

b. \(14497 \div 12\)

a)  

b)  

Definition: Fraction (or a Rational Number)
A rational number is a number that can be written in the fraction form \( \frac{a}{b} \) where ‘a’ is whole number and b is a whole number that is not zero. The top number is called the numerator and the bottom number is called the denominator.

\[
\frac{a}{b} \quad \text{numerator} \\
\frac{b}{a} \quad \text{denominator}
\]

Examples: \( \frac{3}{4}, \frac{5}{7}, \frac{87}{100} \)

A fraction can be used to indicate division.
Since \( \frac{a}{b} \) can be described as \( a \div b \), b cannot be zero since division by zero is not possible--it is undefined.

Example: \( \frac{12}{3} \) is equivalent to \( 12 \div 3 \) which equals 4.

Example 5a. Simplify: \( \frac{28}{4} \)
Answer: \( \frac{28}{4} \) is equivalent to \( 28 \div 4 \) which equals 7.

Example 5b. Simplify: \( \frac{0}{11} \)
Answer: \( \frac{0}{11} \) is equivalent to \( 0 \div 11 \) which equals 0.

Example 5c. Simplify: \( \frac{13}{13} \)
Answer: \( \frac{13}{13} \) is equivalent to \( 13 \div 13 \) which equals 1. (Any number divided by itself equals 1.)

Your Turn Problem #5
Simplify the following.
a) \( \frac{75}{5} \)  
b) \( \frac{21}{21} \)  
c) \( \frac{0}{3} \)
Answer: __________  Answer: __________  Answer: __________
**Useful Technique for Long Division**

When starting a long division problem, it may be beneficial to just write out the products of the divisor and numbers 1 through 9.

**Example:** 4680 ÷ 17

1. First rewrite in long division format.
   
   \[ \begin{array}{c}
   17 \\
   \hline
   4680
   \end{array} \]

2. Write the products of the divisor, 17, and the numbers 1-9 (shown to the right).

   \[ \begin{array}{ccc}
   & \times 1 & \times 2 & \times 3 \\
   \hline
   & 17 & 34 & 51
   \end{array} \]

3. Determine where the first number will be placed in the quotient (above the division box). 17 will not divide into 4 but it will divide into 46. Therefore the first digit of the quotient is written above the 6.

4. Looking at our products to the right, we want to find the product closest to 46 without going over. Answer: 2.

   Write the 2 in the quotient, multiply, then subtract and bring down next digit.

   \[ \begin{array}{c}
   2 \\
   17 \overline{)4680} \\
   - 34 \\
   \hline
   128
   \end{array} \]

5. Do it again. Find the product to the right closest to 128 without going over. Answer: 7

   \[ \begin{array}{c}
   27 \\
   17 \overline{)4680} \\
   - 34 \\
   \hline
   128 \\
   - 119 \\
   \hline
   90
   \end{array} \]

6. Do it again. Find the product to the right closest to 90 without going over. Answer: 5

   \[ \begin{array}{c}
   275 \\
   17 \overline{)4680} \\
   - 34 \\
   \hline
   128 \\
   - 119 \\
   \hline
   90 \\
   - 85 \\
   \hline
   5
   \end{array} \]

\[ \text{Answer: } 275 \text{ R } 5 \]
1.6 Homework: Division of Whole Numbers

Divide the following.

1. \(13 \div 0\)  
2. \(0 \div 9\)  
3. \(0 \div 0\)  
4. \(15 \div 15\)  
5. \(7 \div 7\)  
6. \(141 \div 3\)  
7. \(735 \div 5\)  
8. \(1524 \div 6\)  
9. \(2936 \div 8\)  
10. \(4887 \div 9\)  
11. \(1842 \div 6\)  
12. \(67,921 \div 7\)  
13. \(138 \div 6\)  
14. \(324 \div 6\)  
15. \(756 \div 9\)  
16. \(2,695 \div 4\)  
17. \(45,967 \div 3\)  
18. \(5,648 \div 8\)
1.6 Homework: Division of Whole Numbers cont.

19. \( 1,068 \div 53 \)  
20. \( 51,975 \div 15 \)

21. \( 97,356 \div 42 \)  
22. \( 414 \div 13 \)

23. \( 52,345 \div 24 \)  
24. \( 129,212 \div 16 \)

25. \( 5,324 \div 78 \)  
26. \( 192,211 \div 18 \)

27. \( 567 \div 11 \)  
28. \( 792 \div 24 \)

29. \( 887 \div 18 \)  
30. \( 5461 \div 43 \)

31. \( 264 \div 14 \)  
32. \( 156 \div 30 \)
1.6 Homework: Division of Whole Numbers cont.

33. There are 517 students who are taking a field trip. If each bus can hold 42 students, how many buses will be needed for the field trip?

34. Construction of a fence section requires 8 boards. If you have 260 boards available, how many sections can you build?

35. Mark drove 408 miles on 12 gallons of gas. Find the mileage (in miles per gallon) of Mark’s car.

36. A machine can produce 240 bottles per minute. How long will it take to produce 25,680 bottles?

37. A large container has 735 liters of a liquid. How many 2-liter bottles can be filled from the 735 liters?

Simplify the following fractions.

38. \( \frac{48}{4} \)

39. \( \frac{120}{5} \)

40. \( \frac{27}{27} \)

41. \( \frac{395}{395} \)

42. \( \frac{0}{9} \)

43. \( \frac{0}{12} \)
1.7 Exponents

**Exponential Form**
A form of writing the product of a factor that repeats

Example: \( 2 \cdot 2 \cdot 2 = 2^3 \) or \( 2^3 = 2 \cdot 2 \cdot 2 \)

**Base:** The base is the factor being repeatedly multiplied

**Exponent:** The exponent is a small number written to the right of the base and raised a half space. It indicates the number of bases being multiplied.

![Exponent (or power) - Base \( 2^3 \)]

How to say it.

Exponent: 2 – squared or “to the second power”

3 – cubed or “to the 3rd power”

4 – to the fourth power

etc.

Examples

\( 2^3 \): two cubed or 2 to the 3rd (power)

\( 8^2 \): eight squared or 8 to the 2nd (power)

\( 3^7 \): three to the seventh (power)

The word “power” is often left out.

**Procedure: To evaluate a number in exponential form**

Step 1: Rewrite as repeated multiplication.

Step 2: Multiply the repeated multiplication’s.

Step 3: Perform any other operations
Example 1. a) Evaluate: $3^4$  

b) Evaluate: $5^2$

Solutions:  

$3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$

$5^2 = 5 \cdot 5 = 25$

Your Turn Problem #1

a) Evaluate $5^3$ __________

b) Evaluate $7^2$ __________

Exponents of One

A number with an exponent of “1” is simply equal to the number

Example 2a. Evaluate: $8^1$

Answer: $8^1 = 8$

Exponents of Zero

A number with an exponent of “0” is equal to 1. It doesn’t make too much sense right now. The explanation will come later once we have learned a few algebra tools.

Example 2b. Evaluate: $8^0$

Answer: $8^0 = 1$

Your Turn Problem #2

a) Evaluate $4^1$ __________

b) Evaluate $9^0$ __________
Example 3. Evaluate: $2^5 + 7^2$

Solution: First rewrite each as repeated multiplication $2^5 + 7^2 = 2 \cdot 2 \cdot 2 \cdot 2 + 7 \cdot 7$
Multiply the repeated multiplication’s. $32 + 49$
Perform any other operations. Answer: $81$

Your Turn Problem #3

Evaluate $4^3 + 5^3$

Answer: _________

Example 4. Evaluate: $3^2 \cdot 4^2 \cdot 1^5$

Solution: First rewrite each as repeated multiplication: $3^2 \cdot 4^2 \cdot 1^5 = 3 \cdot 3 \cdot 4 \cdot 4 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1$
Multiply the repeated multiplication’s. $9 \cdot 16 \cdot 1$
Perform any other operations. Answer: $144$

Note: If the base is 1, it doesn’t matter what the exponent is. The result will always be 1.

Your Turn Problem #4

Evaluate: $5^3 \cdot 3^3 \cdot 1^9$

Answer: _________
Evaluating with a base of 10

Example 5a. Evaluate: $10^2$

Solution: First rewrite as repeated multiplication, then multiply.

$$10^2 = 10 \cdot 10$$

Answer: $100$

Example 5b. Evaluate: $10^3$

Solution: First rewrite as repeated multiplication, then multiply.

$$10^3 = \frac{10 \cdot 10 \cdot 10}{100}$$

Answer: $1000$

Note that the pattern. – The number of 10’s being multiplied is the same as the number of zeros in answer. Similarly, the number of zeros in the answer is equal to the exponent.

$$10^1 = 10 \text{ (one zero, same as exponent)}$$

$$10^2 = 100 \text{ (two zeros, same as exponent)}$$

$$10^3 = 1000 \text{ (three zeros, same as exponent)}$$

Using this pattern for $10^8$; the answer would be the number “1” followed by eight “0”s.

$$10^8 = 100000000$$

Then starting from the right, place commas after every third digit.

$$100,000,000$$

Your Turn Problem #5

Evaluate: $10^5$

Answer: ________
Procedure: Evaluating with a base of 10

Step 1: Write the number one, and then insert an amount of zeros equal to the exponent.
Step 2: Perform any other operations and then starting from the right, place commas after every third digit.

Example 6. Evaluate: $7 \cdot 10^5$

Solution: $7 \cdot 10^5$

$\phantom{7} \cdot 100000$ \hspace{1cm} Step 1

Answer: $700\,000$ \hspace{1cm} Step 2

Your Turn Problem #6

Evaluate: $9 \cdot 10^3$

Answer: __________

Example 7. Evaluate: $2^2 \cdot 3^2 \cdot 10^3$

Solution: $2^2 \cdot 3^2 \cdot 10^3$

$\phantom{2^2} \cdot 2 \cdot 3 \cdot 3 \cdot 1000$ \hspace{1cm} Step 1

$\phantom{3^2} \cdot 4 \cdot 9 \cdot 1000$ \hspace{1cm} Step 1

Answer: $36\,000$ \hspace{1cm} Step 2

Your Turn Problem #7

Evaluate: $5^2 \cdot 2^3 \cdot 10^4$

Answer: __________
1.7 Homework: Exponents

Evaluate the following

1. $2^4$  
2. $3^3$  
3. $10^2$

4. $5^3$  
5. $7^3$  
6. $12^2$

7. $6^3$  
8. $3^5$  
9. $11^0$

10. $9^1$  
11. $1^{21}$  
12. $6^0$

13. $8^2$  
14. $3^4$  
15. $7^2$

16. $2^7$  
17. $4^3$  
18. $14^0$

19. $8^2 + 3^3$  
20. $2^5 - 5^2$  
21. $3^4 - 3^3$

22. $2^2 \cdot 3^2 \cdot 4^2$  
23. $1^8 \cdot 3^3 \cdot 2^4$  
24. $1^5 \cdot 2^3 \cdot 9^0$

25. $12 \cdot 10^4$  
26. $2^5 \cdot 10^3$  
27. $3^4 \cdot 2^2 \cdot 10^5$
Order of Operations Agreement

Multiplication and division come before addition and subtraction unless placed within parenthesis. Then there are exponents. When are they to be simplified? To keep it all straight, it is imperative that you memorize the following-- the complete Order of Operations Agreement. Please note the first letter of each step.

Procedure: For Order of Operation

Step 1: Parentheses: perform operations inside parentheses or other grouping symbols (brackets)

Step 2: Exponents: simplify (evaluate) exponential notation expressions

Step 3: Multiply or divide as they appear from left to right

Step 4: Add or subtract as they appear from left to right

When you are asked to “simplify” remember to use the Order of Operations Agreement (PEMDAS). In simplifying, work vertical; for each step recopy the entire problem. For each line of the problem, perform only the step being performed in the Order of Operations.

Restating the Rules:

1. Always simplify what is inside the parentheses or grouping symbols first.
2. If there is a number with an exponent, then evaluate the number with the exponent.
3. Multiplication and Division is performed before Addition and Subtraction.
4. If there is Multiplication or Division next to each other, do whichever is written 1st (left to right).
5. If there is Addition or Subtraction next to each other, do whichever is written 1st (left to right).

Example 1a: Simplify $13 + 2 \cdot 3$

Answer: 19
Example 1b: Simplify $4 + 3(12 - 7)$

Step 1. Parentheses

$4 + 3(5)$

Step 2. Exponents: None

$4 + 15$

Step 3. Multiplication or Division

Step 4. Addition & Subtraction

Answer: $19$

Example 1c: Simplify $12 - 5 + 2$

Step 1. Addition and Subtraction

$7 + 2$

Remember left to right

Answer: $9$

Your Turn Problem #1

a) Simplify $8 + 12 ÷ 3$  

b) Simplify $5 + 3(7 - 2)$  

c) Simplify $24 ÷ 2 · 3$

Answer:_________  

Answer:_________  

Answer:_________

Example 2a: Simplify $2^3 - 5 + 3^2$

Step 1. Parentheses: None

$8 - 5 + 9$

Step 2. Exponents

$3 + 9$

Step 3. Multiplication or Division: None

Answer: $12$

Step 4. Addition & Subtraction: left to right

Example 2b: Simplify $(4 + 2)^2 ÷ 2 · 3$

Step 1. Parentheses

$(6)^2 ÷ 2 · 3$

Step 2. Exponents:

$36 ÷ 2 · 3$

Step 3. Multiplication or Division

$(Left to Right)$

$18 · 3$

Answer: $54$
Your Turn Problem #2

a) Simplify \((5 + 2) + 3(2^3 - 5)\)  
b) Simplify \((2 + 1)^2 - (12 \div 4 \cdot 3)\)

Answer: \(\)  
Answer: \(\)

Example 3a: Simplify \(2 + [8 + 3(5 - 2)]\)  
Step 1. Parentheses: 1st, do inside the inner grouping symbols, parentheses  
\[2 + [8 + 3(3)]\]  
\[2 + [8 + 9]\]  
\[2 + [17]\]  
Answer: \(19\)  
Step 2. Addition & Subtraction: left to right

Example 3b: Simplify \(2 + 5[12 - 4(5 - 3)]\)  
Step 1. Parentheses: 1st inside parentheses.  
\[2 + 5[12 - 4(2)]\]  
Then simplify inside brackets.  
\[2 + 5[12 - 8]\]  
Step 2. Multiplication or Division  
\[2 + 5[4]\]  
Step 3. Addition & Subtraction  
\[2 + 20\]  
Answer: \(22\)

Your Turn Problem #3

a) Simplify \(3 + 4[12 - (11 - 5)]\)  
b) Simplify \(20 - 2[12 \div (6 - 2)]\)

Answer: \(\)  
Answer: \(\)
1.8 Homework: Order of Operations

Simplify the following

1. \( 12 - 3 + 5 \)
2. \( 12 ÷ 3 \cdot 4 \)

3. \( 5 + 3 \times 2 \)
4. \( 18 - 12 ÷ 3 \)

5. \( (12 + 3) ÷ 5 \)
6. \( 18 - 3 \times 2 + 4 \)

7. \( 2^3 ÷ 2 \)
8. \( (7 - 4)^2 \)

9. \( (3 + 1)^4 ÷ 2 \)
10. \( 5^2 - 16 ÷ 2 + 8 \)

11. \( (5 - 3)^3 \times 12 \)
12. \( (10 - 7)^2 + (11 - 9)^3 \)

13. \( 48 ÷ (5 - 4 + 7) \)
14. \( 36 ÷ (2^3 - 7 + 1) \)
1.8 Homework: Order of Operations cont.

15. \(30 \div 6 + 12 \div 2\)  
16. \(36 \div 6 \times 2 + (2^4 - 5^0) \div 3\)

17. \(20 + [12 - (8 - 5)]\)  
18. \(3 + 2[18 - (12 \div 3)]\)

19. \(8 - 3[7 - (3 + 2)]\)  
20. \(5 + 2[(12 + 8) \div 4]\)

21. \((7 + 3)[5 + 2(3 + 2)]\)  
22. \((3 + 4)^2 + 8 \cdot 3\)

23. \(8 + 2\left[(4 + 2)^2 \div 3\right]\)  
24. \(2 + 3\left[24 \div (8 - 5 + 3)\right]\)
1.9 Solving Equations

**Equation**
A sentence with an “=” (equal sign) is called an equation.

**Examples:** $9 + 3 = 12$ ; $15 – 8 = 7$. $x + 5 = 19$, $3 \times N = 51$

An equation is a sentence.

Using the example $9 + 3 = 12$; $9 + 3 = 12$ translates to $9$ and $3$ is $12$. The word “and” is a conjunction.

9 and 3 are the subjects. “Is” is the verb. 12 is the direct object.

**Solution**
An equation may be written with one or more of the numbers not known.

A solution of an equation is a number that would make the sentence true if one of the numbers is not known.

Example: 7 plus what number is 16?

Using symbols: $7 + \square = 16$

The solution to this equation would be 9 because $7 + 9 = 16$.

**Variable**
We usually use a letter instead of a blank box. The letter is called a variable.

Example: $7 + N = 16$

**Solutions of an Equation**
The solution of an equation is a number which when replaced for the variable results in a true sentence. The left side of the equal sign is the same as the right side.

**Solving Equations**
We will begin to solve certain types of equations that are pertinent to the topics of this course. There may be occasions where mentally you can see what the answer is. That’s good. However, try to follow a process if given. When the equations get more complicated, you will have practiced the process and the solution will be easier to find.

**Equations where the Variable is Isolated**
The first equation to consider is an equation where the variable is by itself on one side of the equal sign. The procedure is to perform the operation(s) on the other side of the equal sign.
Example 1a. Solve: \( x = 342 \div 3 \)

Solution: Perform the division on the right side of the “=” sign.

\[
\begin{array}{c|c}
3 & 342 \\
\hline
3 & 114 \\
0 & 04 \\
\hline
12 & \\
12 & \\
0 & \\
\end{array}
\]

Answer: \( x = 114 \)

Example 1b. Solve: \( x = 500 - 192 \)

Solution: Perform the division on the right side of the “=” sign.

\[
\begin{array}{c|c}
5 & 9 \\
\hline
4 & 100 \\
\hline
9 & 302 \\
8 & 0 \\
\end{array}
\]

Answer: \( x = 308 \)

Your Turn Problem #1

Solve the following equations.

a) \( N = 30 \times 500 \)  
   b) \( N = 8460 \div 12 \)

Answer:___________  
Answer:___________

Solving Equations of the form \( x + a = b \)

Let’s look at an example in this form.

\( x + 5 = 8 \)

What number plus 5 equals 8. The answer is 3.

Many students can often visualize what this answer is without showing any work, but we will take advantage of knowing what the answer is and form a procedure.
An equation is a balance. If you add a number to one side, then you must add the same number to the other side to keep the balance. Likewise, if you subtract a number on one side, you must subtract the same number on the other side. Therefore, if we subtract 5 on both sides, we will end up with just $x$ on the left hand side and our 3 on the right hand side. Keep in mind, learning the proper procedures will make your transition to algebra courses much smoother.

**Procedure: To solve an equation of the form $x + a = b$**

1. Subtract ‘$a$’ on both sides
2. Simplify left hand side and the right hand side.

---

**Example 2a.** Solve: $x + 18 = 24$

**Solution:**

\[
egin{align*}
  x + 18 - 18 &= 24 - 18 \\
  x + 0 &= 6 \\
  (18 - 18 &= 0 \text{ and } 24 - 18 = 6)
\end{align*}
\]

**Answer:** $x = 6$

\[(x + 0 = x)\]

**Example 2b.** Solve: $x + 95 = 200$

**Solution:**

\[
egin{align*}
  x + 95 - 95 &= 200 - 95 \\
  x + 0 &= 105 \\
  (95 - 95 &= 0 \text{ and } 200 - 95 = 105)
\end{align*}
\]

\[
\begin{array}{c}
  19 \\
  \underline{20} \\
  \underline{105}
\end{array}
\]

**Answer:** $x = 105$

\[(x + 0 = x)\]

Note: Subtracting the number on the same line is a horizontal method. This process could also have been performed vertically. Choose the method you feel most comfortable with.

**Example 2c.** Solve: $x + 12 = 28$

**Solution:**

\[
egin{align*}
  x + 12 - 12 &= 28 - 12 \\
  x + 0 &= 16 \\
  (12 - 12 &= 0 \text{ and } 28 - 12 = 16)
\end{align*}
\]

**Answer:** $x = 16$
Example 2d. Solve: 9 + N = 30

Solution:

9 + N - 9 = 30 - 9 Subtract 9 on both sides.
N + 9 - 9 = 21 (9 + N is the same as N + 9 ; Comm. Prop)
N + 0 = 21 12 - 12 = 0

Answer: N = 21

Your Turn Problem #2

Solve the following equations.

a) 700 + 134 = 700 + 134
b) x + 46 = 93
  c) 28 + A = 54

a) Answer:__________  b) Answer:__________  c) Answer:__________

Solving Equations of the form a x = b

Let’s look at an example in this form.

3 x = 24

3 times what number equals 24? The answer is 8.

Remember, an equation is a balance. If you multiply a number to one side, then you must multiply the same number to the other side to keep the balance. Likewise, if you divide a number on one side, you must divide the same number on the other side. Therefore, if we divide 3 on both sides, we will end up with just x on the left hand side and our 8 on the right hand side.

We are going to use a fraction notation to indicate division. We used this notation previously in the division section. Let’s review.
Example 3a. Simplify: \( \frac{36}{4} \)

Answer: \( \frac{36}{4} \) is equivalent to \( 36 \div 4 \) which equals 9.

Example 3b. Simplify: \( \frac{6252}{12} \)

Answer: \( \frac{6252}{12} \) is equivalent to \( 6252 \div 12 \) which equals 521.

Example 3c. Simplify: \( \frac{9}{9} \)

Answer: \( \frac{9}{9} \) is equivalent to \( 9 \div 9 \) which equals 1. (Any number divided by itself equals 1.)

Your Turn Problem #3

Simplify the following.

a) \( \frac{72}{4} \)  

Answer: \( \_\_\_\_\_\_\_\_ \)

b) \( \frac{15}{15} \)  

Answer: \( \_\_\_\_\_\_\_\_ \)

c) \( \frac{0}{8} \)  

Answer: \( \_\_\_\_\_\_\_\_ \)

Procedure: To solve an equation of the form \( a \cdot x = b \)

1. Divide by ‘a’ on both sides. We will use fraction notation to indicate division.  
   We will show this step by drawing a line under each side separately for division.  
   Then write the number next to the variable under each line.

2. Simplify left hand side and the right hand side.

Note: The answer will always be a whole number without a remainder. Later, we will cover equations where the answer is not a whole number.
Example 4a. Solve: $3 \cdot x = 24$

Solution:

\[
\frac{3 \cdot x}{3} = \frac{24}{3}
\]

Divide by 3 on both sides.

\[
\frac{3 \cdot x}{3} = \frac{24}{3}
\]

Simplify each side. Any number divided by itself equals 1.

Answer: $x = 8$ (1 \cdot x = x)

Example 4b. Solve: $7 \cdot x = 238$

Solution:

\[
\frac{7 \cdot x}{7} = \frac{238}{7}
\]

Divide by 7 on both sides.

\[
\frac{7 \cdot x}{7} = \frac{238}{7}
\]

Simplify each side. Any number divided by itself equals 1.

Answer: $x = 34$

Your Turn Problem #4

Solve the following equations.

a) $9 \cdot x = 432$

b) $15 \cdot x = 3045$

a) Answer: 

b) Answer:

Equations written “backwards”

Some equations may be written with the variable on the right hand side of the “=” sign.

Example: Solve $15 = x + 3$

Our goal is still the same, we need to get ‘x’ by itself. So always subtract the number next to x on each side.

\[
15 = x + 3
\]

\[
15 - 3 = x + 3 - 3
\]

\[
12 = x \quad \text{This can be written as } x = 12.
\]
Rewriting an equation so that the x is on the left hand side is a valid step given by the symmetric property.

**Symmetric Property:** If \( b = a \), then \( a = b \)

The symmetric property lets us rewrite the equation with the ‘x’ on the left hand side. This is an optional step. Some students find it easier to have x on the left hand side as soon as possible. If you wish to solve the equation without using the symmetric property, it will still work out fine.

**Example 5a.** Solve: \( 45 = x + 7 \)

**Solution:** 

\[
\begin{align*}
    x + 7 &= 45 \\
    -7 &\quad -7 \\
    x + 0 &= 38
\end{align*}
\]

**Answer:** \( x = 38 \)

**Example 5a.** Solve: \( 225 = 3 \cdot x \)

**Solution:** 

\[
\begin{align*}
    3 \cdot x &= 225 \\
    \frac{3 \cdot x}{3} &= \frac{225}{3} \\
    x &= \frac{225}{3} \\
    \frac{3 \cdot x}{3} &= \frac{225}{3} \\
    x &= 75
\end{align*}
\]

**Answer:** \( x = 75 \)

**Your Turn Problem #5**

Solve the following equations.

a) \( 71 = x + 39 \)  

b) \( 2665 = 13 \cdot x \)

a) Answer:__________  

b) Answer:__________
1.9 Homework: Solving Equations

Solve the following equations

1. \( x + 5 = 19 \)  
2. \( x + 86 = 100 \)

3. \( N = 51 - 48 \)  
4. \( N = 72 ÷ 2 \)

5. \( 5 \cdot x = 135 \)  
6. \( 8 \cdot a = 176 \)

7. \( x + 13 = 200 \)  
8. \( x + 77 = 88 \)

9. \( N = 24 \times 19 \)  
10. \( Y = 300 - 124 \)

11. \( 7 \cdot x = 273 \)  
12. \( 12 \cdot c = 600 \)

13. \( x + 27 = 90 \)  
14. \( x + 345 = 2000 \)

15. \( A = 30 - 12 - 5 \)  
16. \( B = 67 - 34 + 15 \)
1.9 Homework: Solving Equations cont.

Solve the following equations

17. \( 17 \cdot x = 1428 \)  
18. \( 14 \cdot b = 14 \)

19. \( 65 + x = 75 \)  
20. \( 43 + x = 115 \)

21. \( 22 = C + 5 \)  
22. \( 148 = x + 52 \)

23. \( 391 = 17 \cdot x \)  
24. \( 896 = 16 \cdot b \)

25. \( 380 \times 200 = N \)  
26. \( 75 - 25 = H \)

27. \( x = 20 - (5 + 8) \)  
28. \( x = 35 - 15 + 5 \)

29. \( x = 5^2 + 3 \times 4 \)  
30. \( x = (97 + 74 + 87) \div 3 \)
1.10 Word Problems using Whole Numbers

Students often ask the question “Why do I need to learn all this math?” The answer is simple. Math was created to solve applications (word problems). The applications will only consist of material that you have learned. Thus far, the material has consisted of operations with whole numbers. Therefore, the applications will consist of operations with whole numbers. Once we cover fractions, the applications will use your knowledge of operations with fractions. Applications will be covered in all of your math classes. Success in solving word problems will be obtained by following a process for solving word problems. Certainly, many students will say “I can do this in my head without showing any steps.” That is great. However, we are also preparing for word problems that you will not be able to do without a procedure to follow.

Reading
Solving word problems begins with reading. We need to comprehend what information is being given and what is being asked. Take your time when you read the problem. Maybe reread it again to make sure you have read it correctly. Write down information while you are reading the problem. If there is a formula that applies, write it down as well. Identify keywords used to indicate the operation.

Let Statement
Once you know what is being asked, write down a “let statement.” Assign a letter to the question being asked. Any letter may be used. We tend to use letters like x, y and n. For example: How many hours will it take to drive to Monterey? The letter (variable) assigned could be H for hours. H = number of hours it will take to drive to Monterey. The “let statement” is stating exactly what is being answered.

Equation
Translate the problem into an equation. Use the variable from the let statement. This step may seem unnecessary at times, however, we are looking at the big picture here. We want to learn the correct process now. Not later.

Solve and Answer
Solve the equation. Then write your answer in sentence. Make sure the answer “makes sense.” If the question is asking for the price of a textbook, and your answer is $8000, it doesn’t make sense. If your answer is $95, that makes sense. Its still expensive, but it makes sense.
General Steps to Solving Word Problems

1. **Read, Write, and Identify.** Read and reread the problem carefully making note of all data (numbers), and keywords. Write down the information. Make a chart, picture, or diagram if possible.

2. **Let Statement.** Identify the question? What is being asked? Choose a symbol for this unknown.

3. **Equation.** Identify the operation to be used (addition, subtraction, multiplication, or division).
   - Translate the problem into an equation using the variable.

4. **Solve.** Solve the equation.

5. **Answer.** Answer the question in a sentence. Make sure proper units ($, feet, hours, books, etc.) are used and the answer “makes sense.”

**Key phrases**
Memorizing the following key phrases will assist you in determining the operation to be performed in a word problem.

**Addition**
Sum, total, increased by, more than, added to, plus
Recall: Addition is commutative. The order does not matter. 5 + 12 = 12 + 5

**Subtraction**
Minus, Subtract, Takeaway, Difference, Subtracted from *, Less than *, Decreased by, How Much More, Taken from, Remains, Deduct, Reduced by, Leftover
Recall: Subtraction is not commutative. 12 – 5 is not the same as 5 – 12.
* For the phrases, “subtracted from” and “less than”, the order must be reversed.
   - 13 subtracted from 20 translates to 20 – 13.
   - Likewise, x subtracted from y translates to y – x. (The 2nd letter or number is written 1st.)

**Multiplication**
Product, Times, Multiplied by, Twice (2 × __), By (10 ft by 12 ft room)
Multiplication is commutative. The order does not matter. 5 × 12 = 12 × 5
Multiplication is also associative: 5 × 7 × 3 = 105 (doesn’t matter which you multiply first.)
   - 35 × 3 = 105 or 5 × 21 = 105

**Division**
Quotient, Divided by/into, Ratio, Per, Split Evenly, how many ____ in ____?
(A total is separated into equal groups)
Example 1. The yearly profit for DR Construction was $78,216 in 2004, $153,917 in 2005, and $85,098 in 2006. What is the total profit for these three years?

Solution:
Step 1. The key word in this problem is “total”.

<table>
<thead>
<tr>
<th>year</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>profit</td>
<td>$78,216</td>
<td>$153,917</td>
<td>$85,098</td>
<td>?</td>
</tr>
</tbody>
</table>

Step 2. Let P = total profit for the three years

Step 3. Equation:  \( P = $78,216 + $153,917 + $85,098 \)

Step 4. Solve:

\[
\begin{array}{c}
78216 \\
153917 \\
+ 85098 \\
\hline
317231 \\
\end{array}
\]

Step 5. Answer question. **The total profit for the three years was $317,231**

Your Turn Problem #1

Laura’s yearly salary is $58,500. Next year, her salary will increase by $7,000. What will her yearly salary be next year?

Data: ____________________________

Key word: __________

Let \( S = ________________________________ \)

Equation: __________________________

Answer: ________________________________
Example 2. In June, the Big Bear Boutique sold $24,760 worth of merchandise, but in July, it sold only $19,458 worth of merchandise. How much more did the boutique sell in June than in July?

Solution: “How much more” indicates subtraction.

Step 1. The key phrase in this problem is “how much more”.

<table>
<thead>
<tr>
<th>Month</th>
<th>Sales</th>
<th>June</th>
<th>July</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$24,760</td>
<td>$19,458</td>
<td></td>
</tr>
</tbody>
</table>

Step 2. Let \( x \) = how much more was sold in June than in July

Step 3. Equation: \( m = 24,760 - 19,458 \)

Step 4. Solve:

\[
\begin{align*}
24760 & \quad - 19458 \\
\hline
      & 5302
\end{align*}
\]

Step 5. Answer: The boutique sold $5,302 more in June than July.

Your Turn Problem #2

The attendance for a concert was 12,329 on Friday and 23,421 on Saturday. How many more people attended on Saturday than on Friday?

Data: ______________________________________________________

Key word:___________

Let \( m \) = __________________________

Equation:__________________________________

Answer:____________________________________________________________
A tip for the operation of multiplication is when a figure is given per some time period (sec., min., hr., day, wk., mo., yr.), and then the problem gives the number of the time period -- for example, the number of hours.

**Example 3.** Marcos agreed to buy a car paying $400 per month for 6 years. How much will he have paid after the 6 years?

**Solution:** keyword: “per” $400 per month for 6 years. 12 months per year.

6 years = ? months. 6 × 12 mo. = 72 months

\[
\frac{400 + 400 + 400 + \ldots + 400}{72 \text{ months}}
\]

Let \( A \) = amount Marcos will have paid after 6 years

\( A = 400 \cdot 72 \)

\( A = 28,800 \)

**Answer:** **Marcos will have paid $28,800 after 6 years**

---

**Your Turn Problem #3**

A Honda holds 17 gallons of gas. If the Honda gets 28 miles per gallon, how far can the Honda go on a full tank of gas?

**Data:** ____________________________________________________

**Key word:**__________

Let \( f \) =______________________________

**Equation:**______________________________

**Answer:**____________________________________________________________
Division defines the operation of finding how many groups of a certain number (the divisor) are contained in another number or amount (the dividend).

Example 4. A person earns $34,500 a year after taxes. How much does this person take home each month?

Solution: The total salary, $34,500 is split evenly into 12 equal groups. Therefore, this is a division problem.

\[
\begin{array}{cccccccc}
\end{array}
\]

12 months
T = take home pay each month
T = 34,500 ÷ 12
T = $2,875

Answer: This person takes home each month $2,875.

Your Turn Problem #4

The receipts from a basketball game totaled $5112. If each ticket was $9, then how many people attended the game?

Data: _____________________________

Keyword:___________

Let ___=_______________________________________________________

Equation:__________________________________

Answer:____________________________________________________________
Average

The average of a set of numbers is the sum of the set numbers, divided by the number of addends.

**Formula for Average:** \( \frac{\text{Sum of a set of numbers}}{\text{number of addends}} \)

**Example 5.** A student received scores of 72, 87, and 54. Find the average of the 3 test scores.

**Solution:** Average Problem, use formula.

Let \( A = \text{average of the three test scores.} \) (Add the scores, then divide by 3.)

\[
A = \frac{72 + 87 + 54}{3} = \frac{213}{3} = 71
\]

**Answer:** The student has an average of 71.

**Your Turn Problem #5**

Lori has 4 math tests with scores of 76, 85, 92, and 91. Find the average of all four tests.

Answer: ________________________________

**Example 6.** On a four-day trip, Valentine drove the following number of miles each day: 240 miles, 360 miles, 118 miles, and 222 miles. What was the average number of miles she drove per day?

**Solution:** Let \( A = \text{average of the number of miles driven per day.} \)

\[
A = \frac{240 + 360 + 118 + 222}{4} = \frac{940}{4} = 253
\]

**Answer:** Valentine drove an average of 253 miles per day.
Your Turn Problem #6
The temperature for a five - day period were as follows: 105°, 101°, 96°, 102°, 106°.
What was the average temperature for the five - day period?

Answer:_________________________________________________________________

Example 7. Raul bought the following items at Home Depot: Three outdoor motion lights for $24 each, one patio table for $125, and 4 chairs for $39 each. If he only has $350 will he have enough money to purchase the items? (assume tax is already included)
If there is enough money, how much money will he still have after the purchase? If he does not have enough money, how much more will he need?

Solution: Write down the data given:
3 lights @ $24
1 table @ $125
4 chairs @ $39 each

What is the total cost of the 8 items?
Let C = The total cost.
C = 3 × $24 + 1 × $125 + 4 × $39
C = $72 + $125 + $156
C = $353

Does Raul have enough money? He only has $350.
350 – 353 is not possible.

Answer: No, Raul does not have enough money. He needs $3 to purchase all of the items.
Your Turn Problem #7
Daniel bought two packs of AA batteries for $7 each, three boxes of cereal for $4 each, and five boxes of pop tarts for $2 each. If he only has $40, will he have enough money to purchase these items (assume tax is already included)? If there is enough money, how much money will he get back in change? If there is not enough how much more will he need?

Answer:_________________________________________________________________

Example 8. Antonio’s salary in 2010 was $42,000. If this is $5000 more than his salary in 2009, what was his salary in 2009?

Solution: Write down the data given:
2010 salary: $42,000
2009 salary is $5000 less than 2010 salary
Let s = Antonio’s salary in 2009
s = 42000 – 5000
s = 37,000

Answer: Antonio’s salary in 2009 was $37,000.

Your Turn Problem #8
During a weekend series between the Dodgers and the Giants, where the Dodgers swept the Giants, the Dodgers scored 26 runs. If the Dodgers scored 8 more runs than did the Giants, how many runs did the Giants score?

Answer:_________________________________________________________________
1.10 Homework: Word Problems using Whole Numbers

1. Cathy spent $364 for tuition, $583 for books, and $35 for parking during one semester. What was the total cost for tuition, books, and parking for that semester?

2. Jacky’s monthly paycheck of $879 was decreased by $175 for tax withholdings. What amount of pay did she receive after taxes?

3. Brian’s checking account has a balance of $575. Brian wrote three checks for $54, $37, and $143. What was the new balance in the checking account?

4. A room contains 47 rows of seats. Each row has 28 seats. How many seats are in the room?

5. Sandra drove 559 miles in her car using 13 gallons of gas. What is the cars miles per gallon (mpg)?

6. Tim borrows $4140. If he arranges to pay off the loan and interest charge in 12 monthly payments, what is the monthly payment?

7. A computer printer can print 40 mailing labels per minute. How many labels can be printed in one hour?

8. Sandra agrees to buy a car paying $260 a month for 5 years. What is the total cost? (Hint: 5 years = 60 months)

9. Lauren agrees to buy a car paying $300 down and $225 a month for 5 years. What is the total cost?
10. There are 568 students who are taking a field trip. If each bus can hold 42 students, how many buses will be needed for the field trip?

11. Construction of a section of fence requires 24 boards. If you have 2600 boards available, how many sections can you build?

12. Brandi earns $647 per week. How much money would she earn in 50 weeks?

13. A student had test scores of 85, 72, 63, 91 and 94. Find the average of the test scores.

14. An employee’s pay rate is $12 per hour. If she works 32 hours a week, how much will she make in two weeks?

15. Julie’s records showed the following test scores in math: 96, 62, 72, and 82. What was her test score average?

16. Juan agrees to buy a car paying $1000 down and $456 a month for 6 years. What is the total cost of the car?

17. A bottle company has 500 bottles of a certain type of liquid. If the machine separates the bottles into 8 bottle cases, how many cases can be created?
1.11 Geometric Applications - Perimeter

Perimeter of a Rectangle

The perimeter of a rectangle is the sum of all of its sides.

If we add all of the sides from the rectangle above, \(7\text{ ft} + 7\text{ ft} + 12\text{ ft} + 12\text{ ft} = 38\text{ ft}\).

Therefore, the perimeter of the rectangle above is 38 feet.

Typically, the length of the shorter side is called the width and the length of the longer side is called the length.

We can label the sides using \(L\) and \(W\) for length and width.

Formulas in math are often used to describe a process. For instance to find the perimeter, we add all of the sides.

\[ L + L + W + W = \text{Perimeter}. \]

\(L + L\) is the same as \(2 \times L\) and \(W + W\) is the same as \(2 \times W\).

So the formula for Perimeter of a Rectangle can be written as:

\[ 2 \times L + 2 \times W = P \quad \text{or} \quad P = 2 \times L + 2 \times W \]

We can also omit the times symbol, and write \(P = 2L + 2W\)

If all these symbols got a little confusing, don’t stress over it. For now, just remember the perimeter means to take the sum of all of its sides.
Perimeter of a Polygon

A polygon is a closed geometric figure with three or more sides. The perimeter of a polygon is the distance around it, or the sum of the lengths of its sides.

Please Note: The sketches in this section are not necessarily to scale.

Example: Find the perimeter of the polygon:
Solution: Add the lengths of all sides.
Perimeter = 5 ft + 10 ft + 4 ft + 14 ft
Answer: 33 ft

Finding a missing side.

In this polygon to the left, there is only one side that is missing. We will label x to show it is an unknown number.

Looking at the horizontal line segments, 4 and 6, we can determine what the bottom horizontal line segment is.

The missing line segment = 4 + 6 = 10

In this polygon to the left, there is one side missing.

Looking at the horizontal line segments, 20 and 15, we can determine what the top horizontal line segment is.
The missing line segment is 20 – 15 = 5

Also, notice the vertical line segments, 3, 9, and 12. The two smaller vertical line segments must add to equal the larger vertical line segment.

In this polygon to the left, there is one side missing.

Looking at the vertical line segments, 17 and 10, we can determine what the top vertical line segment is.
The missing line segment is 17 – 10 = 7

Also, notice the horizontal line segments, 12, 14, and 26. The two smaller horizontal line segments must add to equal the larger horizontal line segment.
Your Turn Problem # 1
Find the missing side.

\[ \begin{array}{c}
9 \\
8 \\
10 \\
12 \\
19 \\
\end{array} \]
Answer:___________

Your Turn Problem # 2
Find the missing side. (labeled x)

\[ \begin{array}{c}
13 \text{ ft} \\
7 \text{ ft} \\
x \\
8 \text{ ft} \\
25 \text{ ft} \\
\end{array} \]
Answer:___________

Your Turn Problem # 3
Find the missing side. (labeled x)

\[ \begin{array}{c}
5 \\
11 \\
8 \\
x \\
7 \\
4 \\
20 \\
\end{array} \]
Answer:___________
Your Turn Problem # 4

Find the missing sides. (labeled x, y, and z)

Answer:______________

Your Turn Problem # 5

Find the missing side. (labeled x and y)

Answer:______________

Once we know the value of all of the sides, we can determine the perimeter by adding up all of the sides.

Example: Find the perimeter of the geometric figure.

The horizontal missing segment is $22 - 18 = 4$.
The vertical missing segment is $12 - 5 = 7$.
To find the perimeter, add all of the sides
$12 + 22 + 5 + 18 + 4 + 7 = 68$ ft
Your Turn Problem #6
Find the perimeter of the geometric figure

Answer:__________

Your Turn Problem #7
Find the perimeter of the geometric figure

Answer:__________
1.11 Homework: Geometric Applications - Perimeter

Find the perimeter of each figure.

1. \( \text{4 ft} \) (square)  

2. \( \text{3 yd} \)  
   \( \text{7 yd} \)

3. \( \text{7 ft} \)  
   \( \text{4 ft} \)  
   \( \text{8 ft} \)

4. \( \text{10 ft} \)  
   \( \text{13 ft} \)  
   \( \text{4 ft} \)

5. \( \text{18 ft} \)  
   \( \text{15 ft} \)  
   \( \text{6 ft} \)  
   \( \text{20 ft} \)

6. \( \text{3 ft} \)  
   \( \text{2 ft} \)  
   \( \text{5 ft} \)  
   \( \text{4 ft} \)  
   \( \text{15 ft} \)

7. \( \text{17 ft} \)  
   \( \text{5 ft} \)  
   \( \text{6 ft} \)  
   \( \text{7 ft} \)  
   \( \text{16 ft} \)

8. \( \text{3 m} \)  
   \( \text{15 m} \)  
   \( \text{8 m} \)  
   \( \text{9 m} \)  
   \( \text{4 m} \)
1.11 Homework: Geometric Applications – Perimeter cont.

9. Find the perimeter of a rectangle if the length is 15 ft and the width is 7 ft.

10. Find the perimeter of a triangle if the lengths of the sides are 17 ft, 12 ft and 27 ft.

11. Find the perimeter of a square if the length of a side is 19 ft.

12. Find the perimeter of a basketball court if it has dimensions of 50 ft by 94 ft.

13. Find the perimeter of a football field which is 120 yards long by 53 yards wide.

14. Find the perimeter of a triangle if the lengths of the sides are 17 ft, 12 ft and 27 ft.

15. Find the perimeter of a square if the length of a side is 19 ft.
### 1.12 Geometric Applications – Area and Volume

**Area of a Rectangle**

The area of a rectangle is length multiplied by width. Area usually measured in square units.

Examples: 1400 ft$^2$ (square feet), 5 mi$^2$ (square miles)

\[
\text{Area of a rectangle} = \text{length} \times \text{width}
\]

**Example 1.** Find the area of the rectangle.

![Rectangle](image)

**Answer:** The area of this rectangle is $12 \text{ ft} \times 5 \text{ ft} = 60 \text{ ft}^2$.

**Your Turn Problem #1**

Find the area of the rectangle.

![Rectangle](image)

**Answer:** __________

**Area of a Parallelogram**

A parallelogram is a 4-sided shape formed by two pairs of parallel lines. Opposite sides are equal in length and opposite angles are equal in measure. To find the area of a parallelogram, multiply the base by the height. The formula is:

\[
\text{Area of a parallelogram} = \text{base} \times \text{height}
\]

Note: parallel lines are lines that are sketched in the exact same direction.

Example of a parallelogram

![Parallelogram](image)
Example 2. Find the area of the parallelogram.

![Parallelogram Diagram]

Answer: The area of this parallelogram is $14 \text{ ft} \times 5 \text{ ft} = 70 \text{ ft}^2$.

Your Turn Problem #2

Find the area of the parallelogram.

![Parallelogram Diagram]

Answer: ____________

Area of a Triangle

The area of a triangle is $(\text{base} \times \text{height}) \div 2$

![Triangle Diagram]

h=height, b=base

Note: There is a reason why we need this formula for Area is $(\text{base} \times \text{height}) \div 2$. Any triangle is half of a parallelogram. Therefore, if we only want half of the parallelogram, we divide the area of a parallelogram by 2.

Example 3. Find the area of the triangle.

![Triangle Diagram]
Your Turn Problem #3

Find the area of the triangle.

The sum of rectangles

The area of some polygons can be found by separating the figure into rectangles. By drawing a vertical line, we can make two rectangles. We can then find the area by finding the area of each individual rectangle A and B.

Example 4. Find the area of the geometric figure.

First, draw a line segment creating two rectangles A and B. (shown above)

To find the area of each rectangle, multiply the length and width

A: We have both length and width of rectangle A
Therefore, the area of rectangle A is $11\text{ ft} \times 6\text{ ft} = 66\text{ ft}^2$.

B: We have both the length and width of the rectangle B.
The width is 7 ft and the length is 12 feet. Do not use the 18 for the length because 18 is the length of B and the width of rectangle A.
Therefore, the area of rectangle B is $12\text{ ft} \times 7\text{ ft} = 84\text{ ft}^2$.

Answer: The total area of the figure is $66\text{ ft}^2 + 84\text{ ft}^2 = 150\text{ ft}^2$
Your Turn Problem #4
Find the area of the geometric figure

Answer: ____________

Example 5. Find the area of the geometric figure.

First, draw a line segment creating two rectangles A and B. (shown above)

To find the area of each rectangle, multiply the length and width

A: We have the length of the rectangle A (17), but not the width.
23 is the length all the way from side to side. So subtract 15 to get the top horizontal line segment.
The width of rectangle A: \(23 - 15 = 8\).

Therefore, the area of rectangle A is \(17 \times 8 = 136 \text{ ft}^2\).

B: We have both length and width of rectangle B

Therefore, the area of rectangle A is \(10 \times 15 = 150 \text{ ft}^2\).

Answer: The total area of the figure is \(136 \text{ ft}^2 + 150 \text{ ft}^2 = 286 \text{ ft}^2\)

Your Turn Problem #5
Find the area of the geometric figure

Answer: ____________
Thus far, we have discussed perimeter and area.

Examples of the units for perimeter are: ft, m, mi, km, …

Examples of units for area usually have the squared symbol: \( ft^2, km^2, mi^2, \ldots \)

The area of a house is 1400 \( ft^2 \) (square feet). A fire has burned 500 \( mi^2 \) (square miles)

**Volume of a rectangular solid.**

**Volume** is how much a three-dimensional shape occupies. The volume of a container is generally understood to be the capacity of the container.

The formula for Volume of a rectangular solid is:

\[
V = B \times H \times W
\]

Where \( B = \) base, \( H = \) height, and \( W = \) width

The order does not matter because multiplication is commutative and associative.

The units for volume will be cubic: \( ft^3, m^3, cm^3, \) cubic feet, cubic meters, cubic centimeters.

Cubic centimeters are also called cc’s.

**Example 6.** Find the volume of the rectangular solid.

\[
\begin{array}{c}
\text{7 ft} \\
\text{7 ft} \\
\text{10 ft}
\end{array}
\]

**Answer:** The volume of this rectangular solid is \( 7 \text{ ft} \times 7 \text{ ft} \times 10 \text{ ft} = 490 \text{ ft}^3 \).

**Your Turn Problem #6**

Find the volume of this rectangular solid.

\[
\begin{array}{c}
\text{15 ft} \\
\text{7 ft} \\
\text{9 ft}
\end{array}
\]

**Answer:** ___________
1.12 Homework: Geometric Applications – Area and Volume

Find the area enclosed by the geometric figure.

1. \[
\text{square} \quad 9 \text{ ft} \]

2. \[
\text{rectangle} \quad 7 \text{ yd} \quad 12 \text{ yd} \]

3. \[
\triangle \quad 8 \text{ ft} \quad 15 \text{ ft} \]

4. \[
\triangle \quad 9 \text{ km} \quad 15 \text{ km} \quad 12 \text{ km} \]

5. \[
\text{rectangle} \quad 11 \text{ ft} \quad 15 \text{ ft} \]

6. \[
\text{rectangle} \quad 9 \text{ mi} \quad 20 \text{ mi} \]

7. \[
\text{irregular shape} \quad 12 \text{ ft} \quad 11 \text{ ft} \quad 30 \text{ ft} \quad 36 \text{ ft} \]

8. \[
\text{irregular shape} \quad 14 \text{ in} \quad 20 \text{ in} \quad 11 \text{ in} \quad 32 \text{ in} \]
1.12 Homework: Geometric Applications – Area and Volume cont.

Find the volume enclosed by the geometric figure.

9. \[ \text{Volume} = 3 \times 3 \times 3 = 27 \text{ cubic feet} \]

10. \[ \text{Volume} = 10 \times 5 \times 6 = 300 \text{ cubic meters} \]

11. Find the area of a rectangle if the length is 18 feet and the width is 15 feet.

12. Find the area of a basketball court if it has dimensions of 50 ft by 94 ft.

13. Find the area of a triangle if the base is 34 feet and the height is 19 feet.

14. A room has an area of 400 square feet. A contractor has given you a bid to tile the room for $9 per square foot (slate tile included). How much will it cost to tile the room?

15. A room measures 28 ft by 16 feet. If a contractor gives you a bid to tile the room for $7 per square foot (porcelain tile included), how much will it cost to tile the room?

16. A room measures 24 ft by 12 feet. If a contractor gives you a bid to tile the room for $11 per square foot (Marble tile included), how much will it cost to tile the room?
2.1 Divisibility Rules

Divisibility

Divisible: A number is said to be “divisible “by a smaller number if the smaller will divide into the larger “evenly” with no remainder.

Example: 36 is divisible by 2 because there is no remainder.  

\[
\begin{array}{c}
2)36 \\
-2 \\
\hline
16 \\
-16 \\
\hline
0
\end{array}
\]

Divisibility Tests

A test of divisibility is a procedure for determining whether a number is divisible by another number by performing a “test” rather than actually dividing.

Divisibility Test for 2

All even numbers are divisible by 2. Even numbers end with either of the digits: 2, 4, 6, 8, or 0.

A number is divisible by 2 if it has a ones digit of 0, 2, 4, 6, or 8 (that is, it has an even ones digit).

Example 1. a) Is 584 divisible by 2?

Answer: Yes, because the last digit is an even number.

To find the quotient, we would still need to divide, but we know there is no remainder.

Example 1. b) Is 39 divisible by 2?

Answer: No, because the last digit is an odd number. If we divided 2 into 39, there would be a remainder.
Your Turn Problem #1
Determine whether the following numbers are divisible by 2. (State yes or no and why.)

a) 90: ________________________________

b) 243: ________________________________

c) 2,518: ________________________________

Divisibility Test for 3

Let’s make a multiplication fact table for 3.

\[
\begin{array}{ccc}
3 \times 0 & = 0 & 3 \times 7 = 21 \\
3 \times 1 & = 3 & 3 \times 8 = 24 \\
3 \times 2 & = 6 & 3 \times 9 = 27 \\
3 \times 3 & = 9 & 3 \times 10 = 30 \\
3 \times 4 & = 12 & 3 \times 11 = 33 \\
3 \times 5 & = 15 & 3 \times 12 = 36 \\
3 \times 6 & = 18 & 3 \times 13 = 39 \\
\end{array}
\]

Notice the patterns.
1. Each result increases by 3.
2. The sum of the digits is: 3, 6, 9 or 12.  
\[(3 \times 6 = 18, \ 1 + 8 = 9)\]
\[(3 \times 8 = 24, \ 2 + 4 = 6)\]

If we kept listing more products with a factor of 3, we would also find that the sum of its digits may be a 15, 18, 21, etc. This gives us the following divisibility test.

A number is divisible by 3 if the sum of its digits is divisible by 3.
Example 2a. Is 57 divisible by 3?
Answer: Yes, 57 is divisible by 3 because $5+7=12$, and 12 is divisible by 3. If we wanted to know how many times 3 divides into 57, we would actually have to divide. This test just tells us 3 will divide evenly into 57.

\[
\begin{array}{c}
19 \\
3)57 \\
-3 \\
27 \\
-27 \\
0
\end{array}
\]

This means that $3 \times 19 = 57$.

Example 2b. Is 621 divisible by 3?
Answer: Yes, 621 is divisible by 3 because $6+2+1=9$, and 9 is divisible by 3.

Example 2c. Is 443 divisible by 3?
Answer: No, 443 is not divisible by 3 because $4+4+3=11$, and 11 is not divisible by 3. If we tried to divide 3 into 443, there would be a remainder.

Your Turn Problem #2
Determine whether the following numbers are divisible by 3. (State yes or no and why.)

a) 171:__________________________________________________________

b) 297:__________________________________________________________

c) 613:__________________________________________________________
Divisibility Test for 5

Let’s make a multiplication fact table for 5.

| 5 × 0 = 0 | 5 × 7 = 35 |
| 5 × 1 = 5 | 5 × 8 = 40 |
| 5 × 2 = 10 | 5 × 9 = 45 |
| 5 × 3 = 15 | 5 × 10 = 50 |
| 5 × 4 = 20 | 5 × 11 = 55 |
| 5 × 5 = 25 | 5 × 12 = 60 |
| 5 × 6 = 30 | 5 × 13 = 65 |

Notice the pattern. Of course each result increases by 5, but also the last digit is either a 0 or a 5.

A number is divisible by 5 if its last digit is either a 5 or 0.

Example 3a. Is 95 divisible by 5?
Answer: Yes, 95 is divisible by 5 because the last digit is a 5.

Example 3b. Is 553 divisible by 5?
Answer: No, 553 is not divisible by 5 because the last digit is not a 0 or a 5. It is a 3.

Example 3c. Is 130 divisible by 5?
Answer: Yes, 130 is divisible by 5 because the last digit is a 0.

Your Turn Problem #3
Determine whether the following numbers are divisible by 5. (State yes or no and why.)

a) 230: __________________________

b) 175: __________________________

c) 352: __________________________
**Divisibility Test for 9**

Let’s make a multiplication fact table for 9.

\[
\begin{align*}
9 \times 1 &= 9 \\
9 \times 2 &= 18 \\
9 \times 3 &= 27 \\
9 \times 4 &= 36 \\
9 \times 5 &= 45 \\
9 \times 6 &= 54 \\
9 \times 7 &= 63 \\
9 \times 8 &= 72 \\
9 \times 9 &= 81 \\
9 \times 10 &= 90
\end{align*}
\]

Notice the patterns.
1. Each result increases by 9.
2. The first digit increase by 1 and the last digit decreases by 1.
3. Each result has a match in reverse order, i.e., 18 and 81, 27 and 72.
4. The sum of the digits equals 9. (1 + 8 = 9, 4 + 5 = 9)

Lets list a few more to observe the pattern.

\[
\begin{align*}
9 \times 11 &= 99 \\
9 \times 12 &= 108 \\
9 \times 13 &= 117 \\
9 \times 14 &= 126 \\
9 \times 32 &= 288 \\
9 \times 73 &= 657
\end{align*}
\]

The sum of the digits will not always equal 9 but it will be a number that 9 divides evenly into such as 18, 27 or 36.

**A number is divisible by 9 if the sum of its digits is divisible by 9 (i.e., 9, 18, 27, 36, …).**
Example 4a. Is 441 divisible by 9?

Answer: Yes, 441 is divisible by 9 because the sum of its digits is divisible by 9.

(4 + 4 + 1 = 9 and 9 is divisible by 9.)

Example 4b. Is 8937 divisible by 9?

Answer: Yes, 8937 is divisible by 9 because the sum of its digits is divisible by 9.

(8 + 9 + 3 + 7 = 27 and 27 is divisible by 9.)

Example 4c. Is 229 divisible by 9?

Answer: No, 229 is not divisible by 9 because the sum of its digits is not divisible by 9.

(2 + 2 + 9 = 13 and 13 is not divisible by 9.)

Your Turn Problem #4
Determine whether the following numbers are divisible by 9. (State yes or no and why.)

a) 5481: _____________________________________________________________________

b) 329: _____________________________________________________________________

c) 504: _____________________________________________________________________

Divisibility Test for 10

Let’s make a multiplication fact table for 10.

\[
\begin{align*}
10 \times 0 &= 0 & 10 \times 7 &= 70 \\
10 \times 1 &= 10 & 10 \times 8 &= 80 \\
10 \times 2 &= 20 & 10 \times 9 &= 90 \\
10 \times 3 &= 30 & 10 \times 10 &= 100 \\
10 \times 4 &= 40 & 10 \times 11 &= 110 \\
10 \times 5 &= 50 & 10 \times 12 &= 120 \\
10 \times 6 &= 60 & 10 \times 13 &= 130 \\
\end{align*}
\]

Notice the pattern. Of course each result increases by 10, but also the last digit is always a 0.
A number is divisible by 10 if its last digit is 0.

**Example 5a.** Is 310 divisible by 10?

**Answer:** Yes, 310 is divisible by 10 because the last digit is a 0.

**Example 5b.** Is 105 divisible by 10?

**Answer:** No, 105 is not divisible by 10 because the last digit is not a 0. It is a 5.

**Your Turn Problem #5**

Determine whether the following numbers are divisible by 10. (State yes or no and why.)

a) 540: ____________________________________________________________________

b) 300: ____________________________________________________________________

c) 255: ____________________________________________________________________

Thus far, we have only discussed divisibility tests for 2, 3, 5, 9 and 10. There are certainly more divisibility rules. Memorizing more may be more complicated than simply dividing to check divisibility.

**Divisibility for 7**

There is a divisibility test for 7 however it is easier to just divide by 7.

**Example 6a.** Is 91 divisible by 7?

```
\[
\begin{array}{c|c}
7 & 91 \\
\hline
7 & -7 \\
-1 & 21 \\
-21 & -21 \\
0 & \\
\end{array}
\]
```

**Answer:** Yes, 91 is divisible by 7. $7 \times 13 = 91$
Example 6b. Is 107 divisible by 7?

Answer: No, 107 is not divisible by 7 because it doesn’t divide evenly into 107.

\[
\begin{array}{c}
& 15 \\
7 \overline{)107} \\
& -7 \\
& 39 \\
& -35 \\
& 4 \\
\end{array}
\]

Example 6c. Is 119 divisible by 7?

Divide 7 into 119 to determine if 119 is divisible by 7.

\[
\begin{array}{c}
& 17 \\
7 \overline{)119} \\
& -7 \\
& 49 \\
& -49 \\
& 0 \\
\end{array}
\]

Answer: Yes, 119 is divisible by 7. \(7 \times 17 = 119\)

Your Turn Problem #6

Determine whether the following numbers are divisible by 7. If it is divisible by 7, show the factors which multiply to equal the given number.

a) 77: __________________________

b) 161: __________________________

c) 131: __________________________
Example 7. Determine which of the numbers 2, 3, 5 and 10 will divide evenly into 225?

Solution: Is 225 divisible by 2? Answer: No, 225 doesn’t end with an even digit.

Is 225 divisible by 3? Answer: Yes, 2 + 2+ 5 =9 and 9 is divisible by 3.

Is 225 divisible by 5? Answer: Yes, 225 ends with a 0 or a 5.

Is 225 divisible by 5? Answer: No, 225 doesn’t end with a 0.

Answer: 225 is divisible by only 3 and 5.

Your Turn Problem #7
Determine which of the numbers 2, 3, 5 and 10 will divide evenly into 710?

Is 710 divisible by 2? Answer: 

Is 710 divisible by 3? Answer: 

Is 710 divisible by 5? Answer: 

Is 710 divisible by 10? Answer: 

Answer: 710 is divisible by: 

2.1 Homework: Divisibility Rules

Determine which of the numbers 2, 3, 5 and 10 will divide exactly into each of the following numbers.

1. 540:________________________
   2:
   3:
   5:
   10:

2. 346:________________________
   2:
   3:
   5:
   10:

3. 621:________________________
   2:
   3:
   5:
   10:

4. 2,690:_______________________
   2:
   3:
   5:
   10:

5. 5,211:_______________________
   2:
   3:
   5:
   10:

6. 4,002:_______________________
   2:
   3:
   5:
   10:

7. 6,732:_______________________
   2:
   3:
   5:
   10:

8. 9,017:_______________________
   2:
   3:
   5:
   10:
2.1 Homework: Divisibility Rules cont.

Determine which of the numbers 2, 3, 5 and 10 will divide exactly into each of the following numbers.

9. 10,950: ________________

   2: ________________
   3: ________________
   5: ________________
   10: ________________

10. 12,579: ________________

   2: ________________
   3: ________________
   5: ________________
   10: ________________

Determine if 7 will divide into the following numbers. If it is divisible by 7, show the factors which multiply to equal the given number.

11. 91: ________________

12. 157: ________________

13. 133: ________________

14. 77: ________________

15. 1,463: ________________
2.1 Homework: Divisibility Rules cont.

Use the following list for problems 16 through 20.
45, 72, 158, 260, 378, 569, 570, 585, 3,541, 4,530, 8,300

16. Which numbers are divisible by 2? ________________________________

17. Which numbers are divisible by 3? ________________________________

18. Which numbers are divisible by 5? ________________________________

19. Which numbers are divisible by 10? ________________________________

20. Which numbers are divisible by 9? ________________________________
2.2 Prime Numbers and Composite Numbers

Terminology: 3 · 4 = 12

The number’s 3 and 4 are called **Factors**. The result, 12, is called the product.

We say that 3 and 4 are factors of 12.

**Prime Numbers (sometimes just called “Primes”)**

A prime number is a number that has only two distinct factors which are 1 and itself. An example of a prime number is 7. Only 1 times 7 equals 7. We could also say that no other number besides 1 and itself divides evenly into a prime number.

A number such as 12 is not a prime number since it has more than two distinct factors.

3 · 4 = 12, and 2 · 6 = 12

**List of the first few prime numbers.**

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, etc.

Note that 1 is not on the list. Since a prime number has two different factors, it does not qualify. So the number 1 is not a prime number.

**Composite Numbers**

Composite Numbers are natural numbers that are not prime numbers.

A composite number has factors other than 1 and itself.

Example: 12 is a composite number because 3 · 4 = 12.

**List of the first few composite numbers.**

4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, etc.

Note that 1 is not on the list. Since a composite number has more than two different factors, it does not qualify. So the number 1 is not a composite number. The number 1 is neither prime nor composite.
Determining if a number is a Prime Number.

A number is not a prime number if it is divisible by a number other than one and itself.

Procedure:
1. Use the divisibility to determine if the number is divisible by 2, 3 or 5.
2. If the number is not divisible by 2, 3 or 5, continue with the prime numbers beginning at 7.
3. Only continue this process until the prime number you are dividing with is a number that when multiplied by itself, the result is larger than the number being tested. Another way of knowing when to stop is “when the number in the quotient is smaller than the divisor.”

If we can not find a prime number that divides evenly into the number being tested, it is prime. If we can find a prime number that divides evenly into the number being tested, it is composite.

Example 1. Is 411 a prime or composite number? (state reason)

Solution: Check sequentially the numbers 2, 3, and 5

Is 411 divisible by 2? No, doesn’t end with an even digit.
Is 411 divisible by 3? Yes, since 4+1+1 = 6 which is divisible by 3.

Answer: 411 is a composite number because it is divisible by 3. (3·137 = 411)

Your Turn Problem #1

Is 237 a prime or composite number? State reason
Example 2. Is 145 a prime or composite number? (state reason)

Solution: Check sequentially the numbers 2, 3, and 5

Is 145 divisible by 2? No, doesn’t end with an even digit.
Is 145 divisible by 3? No, since 1+4+5 = 10 which is not divisible by 3?
Is 145 divisible by 5? Yes since the last digit ends with a 0 or 5.

Answer: 145 is a composite number because it is divisible by 5. \((5 \times 29 = 145)\)

Your Turn Problem #2

Is 135 a prime or composite number? State reason

Example 3. Is 131 a prime or composite number? (state reason)

Solution: Check sequentially the prime numbers.

Is 131 divisible by 2? No, doesn’t end with an even digit.
Is 131 divisible by 3? No, since 1+3+1 = 5 which is not divisible by 3?
Is 131 divisible by 5? No, since the last digit does not end with a 5.
Is 131 divisible by 7? No, there is a remainder. \(\begin{array}{c}
7 \left) \frac{131}{131}
\end{array}\)

\(\begin{array}{c}
- \frac{7}{61}
\end{array}\)

\(\begin{array}{c}
- \frac{56}{5}
\end{array}\)

Is 131 divisible by 11? No, there is a remainder. \(\begin{array}{c}
11 \left) \frac{131}{131}
\end{array}\)

\(\begin{array}{c}
- \frac{11}{21}
\end{array}\)

\(\begin{array}{c}
- \frac{11}{10}
\end{array}\)

Is 131 divisible by 13? No, we multiply 13 by itself, the result is larger than 131. \(13 \times 13 = 169\) which is larger than 131. So we don’t need to try 13. If you divide the 13 into 131, the number in the quotient will be smaller than the 13.

Answer: 131 is a prime number because it is only divisible by 1 and itself.
Your Turn Problem #3
Is 149 a prime or composite number? State reason

Multiples
A multiple of a natural number is a product of that number and any natural number.
In other words, take the number and multiply it by 1, 2, 3, …

Example: Multiples of 6: 6, 12, 18, 24, 30, 36, …
(6×1=6, 6×2=12, 6×3=18, 6×4=24, 6×5=30, etc.)

There is a relationship between prime numbers, composite numbers and multiples. That is, every composite number is a multiple of a prime number.

Example 4a. Complete the list for the first 10 multiples of 2.

2, 4, 6, ____ , ____ , ____ , ____ , ____ , ____ , ____ , ____

Each number above, except the 2, is a composite number.

Example 4b. Complete the list for the first 10 multiples of 13.

13, 26, 39, ____ , ____ , 78 , ____ , 104 , 117 , ____

Each number above, except the 13, is a composite number.

Example 4c. Complete the list for the first 10 multiples of 9.

9, 18, 27, ____ , ____ , ____ , ____ , ____ , ____ , ____ , ____

Each number above, is a composite number (9 is not prime).
2.2 Homework: Prime Numbers and Composite Numbers

1. There are 25 prime numbers less than 100. List all of them.

Identify each number as prime or composite. If composite, show why (what prime number is it divisible by).

2. 11: ___________________ 3. 57: ___________________

4. 23: ___________________ 5. 141: ___________________

6. 18: ___________________ 7. 47: ___________________

8. 91: ___________________ 9. 111: ___________________

10. 235: ___________________ 11. 152: ___________________

12. List the first 10 multiples of 9.

13. List the first 10 multiples of 25.

14. List the first 10 multiples of 11.
2.3 Prime Factorization and Factors

Prime factorization
Prime factorization is the process of rewriting a composite number as a product of only primes.
Prime factorization can only be performed on composite numbers.

Examples: 24 = 2 × 2 × 2 × 3 or 2^3 × 3
35 = 5 × 7
44 = 2 × 2 × 11 or 2^2 × 11

Note: The product of the prime factorization must equal the composite number.

Determining the Prime Factorization of a Composite Number
There are two techniques for finding the prime factorization of a composite number. Please use the method you feel most comfortable with.

Procedure: Tree Method for Determining the Prime Factorization of a Composite Number
Find two factors whose product is the original number. Draw two branches under the original number and write each factor under each branch. Continue the process until the numbers at below each branch is a prime number.

Example 1: Find the prime factorization 24.

The Prime Factorization of 24 is 2 × 2 × 2 × 3 or 2^3 × 3
Note: It does not matter which two factors we start with (just not 1 and the number given). The result will be the same.
Procedure: Division Method for Determining the Prime Factorization of a Composite Number

Divide the given number by the smallest prime number that divides evenly into the number.
Then, directly above, divide the result (answer) by the smallest prime number that divides evenly in that number. Continue this process until you have an answer which is a prime number. The prime numbers obtained is the prime factorization.

Example 1 (again): Find the prime factorization 24.

1. 2 is the smallest prime that divides evenly into 24.
2. Smallest prime that divides evenly into 12: 2 again.
4. The top number is prime, so we’re done.

The Prime Factorization of 24 is $2 \cdot 2 \cdot 2 \cdot 3$ or $2^3 \cdot 3$.

Your Turn Problem #1

Find the prime factorization 60

Answer: ____________________

Listing all of the Factors for a Composite Number

We say that 3 and 4 are factors of 12.

However, the number 12 has other factors: $2 \cdot 6 = 12$, and $1 \cdot 12 = 12$.

So the complete list of factors for 12 is 1, 2, 3, 4, 6, and 12.

Another way of stating what factors is: “Numbers that divide evenly into a given number”. 1, 2, 3, 4, 6, and 12 all divide evenly into 12. The number 5 is not a factor because it does not divide evenly into 12 (there is a remainder).

Once again, there are two methods for listing all of the factors of a given number.
**Procedure for Listing all of the Factors for a Composite Number by Checking Divisibility**

Start by listing the product of any given number, 1 and itself. Continue listing products sequentially beginning with the numbers 2, 3, 4 and so on by determining if it is a factor. When the number you are determining if it is a factor is already listed in the factors, the list is complete.

---

**Example 2.** List *all* the factors of 24.

Check sequentially the numbers 1, 2, 3, and so on, to see if we can form any factorizations.

1 × 24: is the first product. 2 is a factor because the last digit is even. Divide 2 into 24 to find the product \(2 \times 12\). 3 divides evenly into 24 because the sum of its digits is 6 and 3 divides evenly into 6 \(3 \times 8\). 4 divides evenly into 24 \(4 \times 6\). 5 does not divide evenly into 24. 6 does divide evenly into 24, but it was already found.

\[
\begin{array}{c}
24 \\
1 \cdot 24 \\
2 \cdot 12 \\
3 \cdot 8 \\
4 \cdot 6
\end{array}
\]

**Answer:** The factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24

---

**Your Turn Problem #2**

List all the factors of 18

**Answer:** __________________________

---

**Finding all Factors of a Composite Number using the Prime Factorization**

The previous method is definitely a little easier but, only for smaller numbers. This method will help us out with the larger numbers. Once we have the prime factorization of number, we can use it to find the factors of the number. To be a factor:, it must be: 1, the number itself, one of the prime factors, or the product of 2 or more of the prime factors.

Example: The Prime Factorization of 24 is \(2 \times 2 \times 2 \times 3\)

1 is always a factor. 24 the number itself is a factor. The prime numbers are 2 and 3 which are factors. Then the product of 2 or more of the prime factors:

\[
2 \times 2 = 4, \quad 2 \times 3 = 6, \quad 2 \times 2 \times 2 = 8, \quad 2 \times 2 \times 3 = 12
\]

Using this method, we have obtained 1, 2, 3, 4, 6, 8, 12 and 24.
Procedure for Listing all of the Factors for a Composite Number using Prime Factorization

1. Find the prime factorization.
2. 1 and the number itself are factors.
3. Each prime number in the prime factorization is a factor.
4. Products found by all combinations of the prime factors are factors.

Example 3. List all the factors of 135.

Solution:
1. Find the prime factorization (use either method). \[135 = 3 \times 3 \times 3 \times 5\]
2. 1 and 135 are factors.
3. 3 and 5 are factors (each prime number in prime factorization is a factor).
4. Find all products by combinations of 3, 3, 3 and 5.
   \[3 \times 3 = 9, \quad 3 \times 5 = 15, \quad 3 \times 3 \times 3 = 27, \quad 3 \times 3 \times 5 = 45\]

Note: we do not need to multiply all of the numbers. That would equal 135 which is already listed.

Answer: All of the factors of 135 are 1, 3, 5, 9, 15, 27, 45 and 135.

Your Turn Problem #3
List all the factors of 80

Answer: ______________________

Prime Factorization and Factors of a Prime Number
If asked to find the prime factorization of a number which is actually a prime number, simply state the number is a prime number.

If asked to find the factors of a prime number, the factors are only 1 and itself.
Example: The factors of 11 are only 1 and 11.
2.3 Homework: Prime Factorization and Factors

Find the prime factorization of each number.

1. $36 = \underline{\hspace{2cm}}$

2. $24 = \underline{\hspace{2cm}}$

3. $360 = \underline{\hspace{2cm}}$

4. $48 = \underline{\hspace{2cm}}$

5. $100 = \underline{\hspace{2cm}}$

6. $65 = \underline{\hspace{2cm}}$

7. $13 = \underline{\hspace{2cm}}$

8. $55 = \underline{\hspace{2cm}}$

9. $240 = \underline{\hspace{2cm}}$

10. $144 = \underline{\hspace{2cm}}$

11. $147 = \underline{\hspace{2cm}}$

12. $175 = \underline{\hspace{2cm}}$

13. $195 = \underline{\hspace{2cm}}$

14. $154 = \underline{\hspace{2cm}}$

15. $875 = \underline{\hspace{2cm}}$

16. $504 = \underline{\hspace{2cm}}$
2.3 Homework: Prime Factorization and Factors cont.

List all the factors of each of the following numbers

17. 36 = ________________
18. 24 = ________________

19. 15 = ________________
20. 48 = ________________

21. 100 = ________________
22. 65 = ________________

23. 13 = ________________
24. 55 = ________________

25. 64 = ________________
26. 6 = ________________

27. 90 = ________________
28. 60 = ________________

29. 30 = ________________
30. 56 = ________________
2.4 The GCF (Greatest Common Factor)

Greatest Common Factor (GCF)
The greatest common factor of two or more numbers is the largest factor that the numbers have in common. Another way of stating the GCF is: the largest number that divides evenly into the numbers given.

For example: Find the GCF of 30 and 45.

Solution: We could list all of the factors of each number. Then the GCF would be the largest number on both lists.

30 = 1, 2, 3, 5, 6, 10, 15, 10
45 = 1, 3, 5, 9, 15, 45

Therefore the GCF is 15 because it is the largest number common to both lists of factors.

We also stated that the GCF is the largest number that divides evenly into both numbers. Since 30 and 45 are divisible by 3, 5, and 15 and the greatest of those numbers is 15, then the GCF = 15.

Instead of listing all of the factors, or trying to come up with the numbers that divide into the numbers given, we have a better technique for finding the GCF using Prime Factorization.

Procedure for finding the GCF using Prime Factorization

Step 1. Find the prime factorization of each number. If the number is prime, simply write down the same prime number after the “=” sign.

Step 2. Circle any prime number that appears in all prime factorization lists. It is possible that two or more of any prime number can be common to all rows.

Step 3. Multiply the common primes from one row together. This is the GCF. If there is only one prime factor in common, then there is nothing to multiply and that prime factor is the GCF. If there are no primes in common, then the GCF is 1. (1 is always a factor of any number.)

Example 1. Find the GCF of 12 and 18.

Step 1. Find the prime factorization of each number.

\[
12 = 2 \times 2 \times 3 \\
18 = 2 \times 3 \times 3
\]

Step 2. Circle any prime that appears in both rows.

Step 3. Multiply the common primes together from one row.

The GCF = 2 \times 3 = 6

(Tree method is also fine.)
Your Turn Problem #1

Find the GCF of 28 and 42.

\[
28 = \quad 42 = \quad \text{GCF} =
\]

Example 2. Find the GCF of 54, 90 and 108.

**Step 1.** Find the prime factorization of each number.

<table>
<thead>
<tr>
<th>Number</th>
<th>Prime Factorization</th>
</tr>
</thead>
<tbody>
<tr>
<td>54</td>
<td>(2 \cdot 3 \cdot 3 \cdot 3)</td>
</tr>
<tr>
<td>90</td>
<td>(2 \cdot 3 \cdot 3 \cdot 5)</td>
</tr>
<tr>
<td>108</td>
<td>(2 \cdot 2 \cdot 3 \cdot 3 \cdot 3)</td>
</tr>
</tbody>
</table>

**Step 2.** Circle any prime that appears in all three rows.

**Step 3.** Multiply the common primes together from one row.

The GCF = \(2 \cdot 3 \cdot 3 = 18\)

Your Turn Problem #2

Find the GCF of 88, 132 and 220.

\[
88 = \quad 132 = \quad 220 = \quad \text{GCF} =
\]
Example 3. Find the GCF of 8 and 15.

**Step 1.** Find the prime factorization of each number.

\[
\begin{align*}
8 &= 2 \cdot 2 \cdot 2 \\
15 &= 3 \cdot 5
\end{align*}
\]

**Step 2.** Circle any prime that appears in all three rows. (none)

**Step 3.** If there are no primes in common, then the GCF is 1.

**The GCF = 1**

---

Your Turn Problem #3

Find the GCF of 4, 9 and 25.

\[
\begin{align*}
9 &= \\
25 &= \\
\text{GCF} &=
\end{align*}
\]

Example 4. Find the GCF of 11, 22 and 44.

**Step 1.** Find the prime factorization of each number.

\[
\begin{align*}
11 &= 11 \\
22 &= 2 \cdot 11 \\
44 &= 2 \cdot 2 \cdot 11
\end{align*}
\]

**Step 2.** Circle any prime that appears in all three rows.

**Step 3.** If there is only one prime factor in common, then there is nothing to multiply and that prime factor is the GCF.

**The GCF = 11**

---

Your Turn Problem #4

Find the GCF of 13, 39 and 52.

\[
\begin{align*}
13 &= \\
39 &= \\
78 &= \\
\text{GCF} &=
\end{align*}
\]
2.4 Homework: The GCF (Greatest Common Factor)

Find the greatest common factor (GCF) for each of the following groups of numbers

1. 12 and 18
   12 = 
   18 = 
   GCF =

2. 15 and 25
   15 =
   25 =
   GCF =

3. 22 and 14
4. 10 and 30

5. 21 and 28
6. 12, 36 and 60

7. 26, 39, and 52
8. 55, 66, and 77

9. 25, 75, and 150
10. 12 and 25

11. 92 and 138
12. 38 and 57
2.5  The LCM (Least Common Multiple)

**Multiples**

A multiple of a natural number is a product of that number and any natural number. In other words, take the number and multiply it by 1, 2, 3, …

Example: Multiples of 6: 6, 12, 18, 24, 30, 36, …

\((6 \times 1=6, \ 6 \times 2=12, \ 6 \times 3=18, \ 6 \times 4=24, \ 6 \times 5=30, \ \text{etc.})\)

**Least Common Multiple**

The least common multiple, or LCM, of two natural numbers is the smallest number that is a multiple of both numbers.

Example: Observe the list of multiples for the following numbers.

6: 6, 12, 18, 24, 30, 36, 42, 48, 54 …
8: 8, 16, 24, 32, 40, 48, 56, 64, …

The **LCM** is the smallest number that appears in both lists. (Least Common Multiple)

24 is the smallest number that appears in both lists, therefore it is the LCM. There are other common multiples such as 48, but 24 is the lowest common multiple.

Finding a LCM is part of the process to adding and subtracting fractions with unlike denominators so it is very important. Writing down a list of multiples is sufficient method as long as the numbers are relatively small. What if the numbers are 84 and 90? Then, making a list of multiples is a little tedious. Fortunately, we have a better method.

**Prime Factorization Method for Finding the LCM (Least Common Multiple)**

**Procedure:** Finding the LCM using the Prime Factorization Method

**Step 1.** Find the prime factorization of each number. You can use either method. Use either the prime factorization tree or the division method.

**Step 2.** Find the prime factors that appear in either factorization.

**Step 3.** The LCM = product of these prime factors, writing each prime factor the greatest number of times that it occurs in any one of the prime factorizations.
**Restating Step 3 from the procedure.**

Start with the smallest prime number in either factorization.

Ask the following questions to yourself: “Which has the most of that prime number?” and “How many times does it occur in that row?” Write that prime number that same number of times. Then continue on with the next smallest prime number.

For example: Suppose the top row has three 7’s and the bottom row has five 7’s.

Question: Which row has the most? Answer: The bottom row. How many? It has five.

The LCM will have five 7’s.

What if both rows have the same amount of a prime number? Then choose the number of primes from the top row (same as the number of primes from the bottom row).

Common Mistake: Do not take the same prime factor from two different rows. Your LCM will be too big.

**Example 1.** Find the LCM of 15 and 24.

\[
\begin{align*}
15 & \quad 24 \\
3 & \quad 5 \\
2 & \quad 2 \\
\end{align*}
\]

Step 1. Find the prime factorization of each number.

Step 2. Write “LCM=” below the prime factorization of each number.

Consider each prime number. Write it the greatest number of times it occurs in any one factorization.

\[
\begin{align*}
15 & = 3 \cdot 5 \\
24 & = 2 \cdot 2 \cdot 2 \cdot 3 \\
\text{LCM} & = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \\
& = 120
\end{align*}
\]

Now multiply. This is the LCM.

**Your Turn Problem #1**

Find the LCM of 35 and 90

\[
\begin{align*}
35 & = \\
90 & = \\
\text{LCM} & = 
\end{align*}
\]
Example 2. Find the LCM of 12, 30, and 70.

\[
\begin{array}{ccc}
12 & 30 & 70 \\
2 & 3 & 2 & 5 & 10 \\
2 & 3 & 5 & 7 & 10 \\
\end{array}
\]

Step 1. Find the prime factorization of each number.

Step 2. Write “LCM=” below the prime factorization of each number.

Consider each prime number. Write it the greatest number of times it occurs in any one factorization.

2: it occurs twice.
3: it occurs once.
5: it occurs once.
7: it occurs once.

Now multiply. This is the LCM.

\[12 = 2 \times 2 \times 3\]
\[30 = 2 \times 3 \times 5\]
\[70 = 2 \times 5 \times 7\]

LCM = \[2 \times 2 \times 3 \times 5 \times 7 = 420\]

Your Turn Problem #2
Find the LCM of 24, 150 and 240

\[
\begin{array}{c}
24 = \\
150 = \\
240 = \\
\end{array}
\]

LCM =

Example 3. Find the LCM of 7 and 15.

\[
\begin{array}{c}
7 \\
15 \\
\end{array}
\]

Step 1. Find the prime factorization of each number.

Step 2. Write “LCM=” below the prime factorization of each number.

Consider each prime number. Write it the greatest number of times it occurs in any one factorization.

3: it occurs once.
5: it occurs once.
7: it occurs once.

Now multiply. This is the LCM.

\[
\begin{array}{c}
7 = \\
15 = 3 \times 5 \\
\end{array}
\]

LCM = \[3 \times 5 \times 7 = 105\]
Your Turn Problem #3
Find the LCM of 11 and 25

11 =
25 =
LCM =

Example 4. Find the LCM of 39, 45, and 65.

Step 1. Find the prime factorization of each number.

Step 2. Write “LCM=” below the prime factorization of each number.

Consider each prime number. Write it the greatest number of times it occurs in any one factorization.

3: it occurs twice.
5: it occurs once.
13: it occurs once.

Now multiply. This is the LCM.

Your Turn Problem #4
Find the LCM of 10, 46 and 115

10 =
46 =
115 =
LCM =
2.5 Homework: The LCM (Least Common Multiple)

1. 9, 15

2. 18, 24, 27

3. 35, 45

4. 15, 19, 24

5. 18, 24

6. 8, 20

7. 16, 20

8. 15, 24

9. 35, 75

10. 12, 18, 24

11. 24, 36

12. 84, 90

13. 35, 45, 63

14. 30, 48

15. 9, 12, and 18

16. 15, 20 and 6

17. 12, 24, and 15

18. 27, 24, and 36

19. 15, 33, and 55

20. 12, 18, and 24
1.1 Answers: Place Value of Whole Numbers

YT#1  a) 7 ten thousands  b) 7 tens  c) 7 billions

YT#2  a) Nine thousand, two hundred seven
       b) Three million, four hundred twenty-nine thousand, seven hundred eighteen

YT#3  a) 135,402  b) 4,000,408,046

1. hundreds  2. hundred thousands  3. hundred millions  4. ten billions
5. thousands  6. tens  7. twenty-one  8. nine hundred forty-one
9. three thousand, five hundred one  10. ninety-three thousand, eight hundred eighty
11. thirty-four million, fifty-eight thousand, twelve
12. seven trillion, twelve billion, thirty thousand, eight
13. 47  14. 327  15. 5,082  16. 135,402  17. 8,000,000,007,000
18. 2,400,011  19. 43,904  20. 8,000,200,080

1.2 Answers: Rounding and Ordering of Whole Numbers

YT#1  YT#2  YT#3  YT#4  YT#5
a) 30  a) 73,000  a) 45,000  a) 250,000  a) >
b) 860  b) 408,300  b) 0  b) 3,440,00  b) >
c) 6,800  c) 0  c) 1000  
1. 350  2. 3,510  3. 13,520  4. 2,400  5. 0
6. 440  7. 76,430  8. 571,600  9. 111,300  10. 700
11. 3,600  12. 14,000  13. 30,000  14. 700  15. 39,700
16. 711,900  17. 43,000  18. 100  19. 7,000  20. 18,000
21. 20,000  22. 0  23. 1,000  24. 3,000  25. 773,413,000
26. 10,000  27. 7,001,000  28. 180,000  29. 45,900,000  30. 227,000,000
31. 340,000,000  32. <  33. >  34. <  35. >
36. >  37. <
### 1.3 Answers: Addition of Whole Numbers

#### YT#1
- **a)** Additive Identity
- **b)** Commutative Property of Addition
- **c)** Associative Property of Addition

<p>| | | | |</p>
<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>7.</td>
<td>17</td>
<td>8.</td>
<td>26</td>
</tr>
<tr>
<td>11.</td>
<td>113</td>
<td>12.</td>
<td>143</td>
</tr>
<tr>
<td>15.</td>
<td>477</td>
<td>16.</td>
<td>1,603</td>
</tr>
<tr>
<td>19.</td>
<td>1,814</td>
<td>20.</td>
<td>6,417</td>
</tr>
<tr>
<td>23.</td>
<td>158,450</td>
<td>24.</td>
<td>640,070</td>
</tr>
<tr>
<td>27.</td>
<td>40</td>
<td>28.</td>
<td>6176</td>
</tr>
<tr>
<td>31.</td>
<td>556</td>
<td>32.</td>
<td>2,460 miles</td>
</tr>
</tbody>
</table>

#### YT#3
- **1.** Additive Identity
- **2.** Associative Property of Addition
- **3.** Associative Property of Addition
- **4.** Additive Identity

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>5.</td>
<td>113</td>
<td>6.</td>
<td>143</td>
</tr>
<tr>
<td>9.</td>
<td>39</td>
<td>10.</td>
<td>60</td>
</tr>
<tr>
<td>13.</td>
<td>79</td>
<td>14.</td>
<td>182</td>
</tr>
<tr>
<td>17.</td>
<td>806</td>
<td>18.</td>
<td>1,622</td>
</tr>
<tr>
<td>21.</td>
<td>139,251</td>
<td>22.</td>
<td>2,448</td>
</tr>
<tr>
<td>25.</td>
<td>6,207</td>
<td>26.</td>
<td>57,559</td>
</tr>
<tr>
<td>29.</td>
<td>172</td>
<td>30.</td>
<td>716</td>
</tr>
<tr>
<td>33.</td>
<td>$982</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 1.4 Answers: Subtraction of Whole Numbers

#### YT#1
- **a)** 34
- **b)** 3

#### YT#2
- **a)** 258
- **b)** 15,678
- **c)** 80,173

#### YT#3
- **11,092**

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
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<td>1.</td>
<td>13</td>
<td>2.</td>
<td>73</td>
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<td>28</td>
<td>4.</td>
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<tr>
<td>5.</td>
<td>6</td>
<td>6.</td>
<td>38</td>
<td>7.</td>
<td>14</td>
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<tr>
<td>9.</td>
<td>35</td>
<td>10.</td>
<td>44</td>
<td>11.</td>
<td>419</td>
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<td>13.</td>
<td>46</td>
<td>14.</td>
<td>28</td>
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<td>16.</td>
</tr>
<tr>
<td>21.</td>
<td>1746</td>
<td>22.</td>
<td>2,683</td>
<td>23.</td>
<td>5,663</td>
<td>24.</td>
</tr>
<tr>
<td>25.</td>
<td>67</td>
<td>26.</td>
<td>649</td>
<td>27.</td>
<td>7,590</td>
<td>28.</td>
</tr>
<tr>
<td>29.</td>
<td>5,785</td>
<td>30.</td>
<td>422</td>
<td>31.</td>
<td>555</td>
<td>32.</td>
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<tr>
<td>33.</td>
<td>74,395</td>
<td>34.</td>
<td>3</td>
<td>35.</td>
<td>10</td>
<td>36.</td>
</tr>
<tr>
<td>37.</td>
<td>352</td>
<td>38.</td>
<td>646</td>
<td>39.</td>
<td>556</td>
<td>40.</td>
</tr>
<tr>
<td>41.</td>
<td>13,846</td>
<td>42.</td>
<td>4</td>
<td>43.</td>
<td>52</td>
<td>44.</td>
</tr>
<tr>
<td>45.</td>
<td>$513</td>
<td>46.</td>
<td>$341</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1.5 Answers: Multiplication of Whole Numbers

**YT #1**
- a) Multiplicative Identity Property
- b) Commutative Property of Multiplication
- c) Distributive Property
- d) Multiplication Property of Zero
- e) Associative Property of Multiplication

**YT #2**
- a) 216
- b) 2850

**YT #3**
- a) 420
- b) 13,320

**YT #4**
- a) 63,000
- b) 1,400,000

1. Commutative Property of Multiplication
2. Multiplicative Identity Property
3. Multiplication Property of Zero
4. Commutative Property of Multiplication
5. Associative Property of Multiplication
6. Multiplication Property of Zero
7. Distributive Property
8. Multiplicative Identity Property
9. Associative Property of Multiplication
10. 96
11. 105
12. 192
13. 252
14. 1541
15. 2448
16. 6080
17. 1872
18. 1748
19. 682
20. 1768
21. 8,304
22. 11,736
23. 40,638
24. 26,208
25. 22,248
26. 263,160
27. 94,500
28. 561,176
29. 177,600
30. 609,364
31. 1,431,565
32. 1,409,382
33. 1,040
34. 267,716
35. 1,437,696
36. 1,500,000
37. 720,000
38. 280,000
39. 8,400,000
40. 84
41. 21,504
42. 375 cal.
43. $1,890
44. 408 seats
45. 60 apt.
46. 14,000 Koi
47. 432 computers
48. 6570 days

1.6 Answers: Division of Whole Numbers

**YT #1**
- a) 1
- b) 34

**YT #2**
- a) undefined
- b) indeterminate
- c) 0

**YT #3**
- a) 532
- b) 503

**YT #4**
- a) 71 R 1
- b) 1208 R 1

**YT #5**
- a) 15
- b) 1
- c) 0

1. undefined
2. 0
3. Indeterminate
4. 1
5. 1
6. 47
7. 147
8. 254
9. 367
10. 543
11. 307
12. 9703
13. 23
14. 54
15. 84
16. 653 r 3
17. 15,322 r 1
18. 706
19. 20 r 8
20. 3,465
21. 2318
22. 31 r 11
23. 2181 r 1
24. 8075 r 12
25. 68 r 20
26. 10,678 r 7
27. 51 r 6
28. 33
29. 49 r 5
30. 127
31. 18 r 12
32. 5 r 6
33. 13 buses
34. 32 sections
35. 34 mpg
36. 107 minutes
37. 367 bottles
38. 12
39. 2
40. 1
41. 1
42. 0
43. 0
1.7 Answers: Exponents

<table>
<thead>
<tr>
<th>YT#1</th>
<th>a) 125</th>
<th>b) 49</th>
<th>YT#2</th>
<th>a) 4</th>
<th>b) 1</th>
<th>YT#3</th>
<th>189</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>16</td>
<td></td>
<td>2.</td>
<td>27</td>
<td></td>
<td>3.</td>
<td>100</td>
</tr>
<tr>
<td>4.</td>
<td>125</td>
<td></td>
<td>5.</td>
<td>343</td>
<td></td>
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</tr>
<tr>
<td>6.</td>
<td>144</td>
<td>7.</td>
<td>216</td>
<td>8.</td>
<td>243</td>
<td>9.</td>
<td>1</td>
</tr>
<tr>
<td>10.</td>
<td>9</td>
<td>13.</td>
<td>64</td>
<td>14.</td>
<td>81</td>
<td>15.</td>
<td>49</td>
</tr>
<tr>
<td>16.</td>
<td>128</td>
<td>17.</td>
<td>64</td>
<td>18.</td>
<td>1</td>
<td>19.</td>
<td>27</td>
</tr>
<tr>
<td>20.</td>
<td>7</td>
<td>22.</td>
<td>576</td>
<td>23.</td>
<td>432</td>
<td>24.</td>
<td>8</td>
</tr>
<tr>
<td>25.</td>
<td>120,000</td>
<td>27.</td>
<td>32,400,000</td>
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</table>

1.8 Answers: Order of Operations

<table>
<thead>
<tr>
<th>YT#1</th>
<th>a) 12</th>
<th>b) 20</th>
<th>c) 36</th>
<th>YT#2</th>
<th>a) 16</th>
<th>b) 0</th>
<th>YT#3</th>
<th>a) 27</th>
<th>b) 14</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>14</td>
<td>2.</td>
<td>16</td>
<td>3.</td>
<td>11</td>
<td>4.</td>
<td>14</td>
<td>5.</td>
<td>3</td>
</tr>
<tr>
<td>6.</td>
<td>16</td>
<td>7.</td>
<td>9</td>
<td>8.</td>
<td>128</td>
<td>9.</td>
<td>25</td>
<td>10.</td>
<td>96</td>
</tr>
<tr>
<td>11.</td>
<td>17</td>
<td>12.</td>
<td>1</td>
<td>13.</td>
<td>64</td>
<td>14.</td>
<td>81</td>
<td>15.</td>
<td>49</td>
</tr>
<tr>
<td>16.</td>
<td>18</td>
<td>17.</td>
<td>1</td>
<td>18.</td>
<td>27</td>
<td>19.</td>
<td>29</td>
<td>20.</td>
<td>7</td>
</tr>
<tr>
<td>21.</td>
<td>150</td>
<td>22.</td>
<td>73</td>
<td>23.</td>
<td>32</td>
<td>24.</td>
<td>14</td>
<td></td>
<td></td>
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</tbody>
</table>

1.9 Answers: Solving Equations

<table>
<thead>
<tr>
<th>1.</th>
<th>x = 14</th>
<th>2.</th>
<th>x = 14</th>
<th>3.</th>
<th>N = 3</th>
<th>4.</th>
<th>N = 36</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.</td>
<td>x = 27</td>
<td>6.</td>
<td>a = 22</td>
<td>7.</td>
<td>x = 187</td>
<td>8.</td>
<td>x = 11</td>
</tr>
<tr>
<td>17.</td>
<td>x = 84</td>
<td>18.</td>
<td>b = 1</td>
<td>19.</td>
<td>x = 10</td>
<td>20.</td>
<td>x = 72</td>
</tr>
<tr>
<td>21.</td>
<td>C = 17</td>
<td>22.</td>
<td>x = 96</td>
<td>23.</td>
<td>x = 23</td>
<td>24.</td>
<td>b = 56</td>
</tr>
<tr>
<td>27.</td>
<td>x = 37</td>
<td>29.</td>
<td>x = 86</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1.10 Answers: Word Problems using Whole Numbers

<table>
<thead>
<tr>
<th>YT#1</th>
<th>$65,500</th>
<th>YT#2</th>
<th>11,092</th>
<th>YT#3</th>
<th>476 miles</th>
<th>YT#4</th>
<th>568 people</th>
</tr>
</thead>
<tbody>
<tr>
<td>YT#5</td>
<td>86</td>
<td>YT#6</td>
<td>The average temperature for the five - day period was 102°.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>YT#7</td>
<td>He has enough money. He will get back $4.</td>
<td>YT#8</td>
<td>18 runs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.</td>
<td>$982</td>
<td>2.</td>
<td>$704</td>
<td>3.</td>
<td>$341</td>
<td>4.</td>
<td>1316 seats</td>
</tr>
<tr>
<td>5.</td>
<td>43 mpg</td>
<td>6.</td>
<td>$345</td>
<td>7.</td>
<td>2400 labels</td>
<td>8.</td>
<td>$15,600</td>
</tr>
<tr>
<td>9.</td>
<td>$13,800</td>
<td>10.</td>
<td>14 buses</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11.</td>
<td>108 sections</td>
<td>12.</td>
<td>$32,350</td>
<td>13.</td>
<td>81</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14.</td>
<td>$768</td>
<td>15.</td>
<td>78</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16.</td>
<td>$33,832</td>
<td>17.</td>
<td>62 cases</td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>
### 1.11 Answers: Geometric Applications - Perimeter

<table>
<thead>
<tr>
<th>YT#</th>
<th>YT#</th>
<th>YT#</th>
<th>YT#</th>
</tr>
</thead>
<tbody>
<tr>
<td>YT#1</td>
<td>20</td>
<td>YT#2</td>
<td>12 ft</td>
</tr>
<tr>
<td>YT#5</td>
<td>16 and 10</td>
<td>YT#6</td>
<td>62 ft</td>
</tr>
<tr>
<td>1.</td>
<td>16 ft</td>
<td>2.</td>
<td>20 yd</td>
</tr>
<tr>
<td>5.</td>
<td>76 ft</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>42 ft</td>
<td>7.</td>
<td>66 ft</td>
</tr>
<tr>
<td>10.</td>
<td>68 ft</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11.</td>
<td>44 ft</td>
<td>12.</td>
<td>288 ft</td>
</tr>
<tr>
<td>15.</td>
<td>76 ft</td>
<td></td>
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</tbody>
</table>

### 1.12 Answers: Geometric Applications – Area and Volume

<table>
<thead>
<tr>
<th>YT#</th>
<th>YT#</th>
<th>YT#</th>
<th>YT#</th>
</tr>
</thead>
<tbody>
<tr>
<td>YT#1</td>
<td>154 m²</td>
<td>YT#2</td>
<td>77 m²</td>
</tr>
<tr>
<td>YT#5</td>
<td>328 ft²</td>
<td>YT#6</td>
<td>945 ft³</td>
</tr>
<tr>
<td>1.</td>
<td>81 ft²</td>
<td>2.</td>
<td>84 yd²</td>
</tr>
<tr>
<td>5.</td>
<td>165 ft²</td>
<td>6.</td>
<td>180 m²</td>
</tr>
<tr>
<td>9.</td>
<td>27 ft³</td>
<td>10.</td>
<td>300 m³</td>
</tr>
<tr>
<td>13.</td>
<td>323 ft²</td>
<td>14.</td>
<td>$3600</td>
</tr>
</tbody>
</table>

### 2.1 Answers: Divisibility Rules

YT#1  

a) Yes. 90 ends with an even digit.

b) No. 243 ends with an odd digit.

c) Yes. 2,518 ends with an even digit.

YT#2  
a) Yes. 1 + 7 + 1 = 9 and 9 is divisible by 3.

b) Yes. 2 + 9 + 7 = 18 and 18 is divisible by 3.

c) No. 6 + 1 + 3 = 10 and 10 is not divisible by 3.

YT#3  
a) Yes. 230 ends with a 0 or a 5.

b) Yes. 175 ends with a 0 or a 5.

c) No. 352 does not end with a 0 or a 5.

YT#4  
a) Yes. 5 + 4 + 8 + 1 = 18 and 18 is divisible by 9.

b) No. 3 + 2 + 9 = 14 and 14 is not divisible by 9.

c) Yes. 5 + 0 + 4 = 9 and 9 is divisible by 9.

YT#5  
a) Yes. 540 ends with a 0.

b) Yes. 300 ends with a 0.

c) No. 255 does not end with a 0.

YT#6  
a) Yes. 7 divides evenly into 77. 7 × 11 = 77.

b) Yes. 7 divides evenly into 161. 7 × 23 = 161.

c) No. 7 does not divide evenly into 131.
2.1 Answers: Divisibility Rules cont.

YT#7  710 is divisible by 2, 5 and 10.

1. 2, 3, 5, 10  2. 2  3. 3  4. 2, 5, 10  5. 3
6. 2, 3  7. 2, 3  8. none  9. 2, 3, 5, 10  10. 3
11. yes  12. no  13. yes  14. yes  15. yes

16. 72, 158, 260, 378, 570, 4,530, 8,300  17. 45, 72, 378, 570, 585, 4,530
18. 45, 260, 570, 585, 4,530, 8,300  19. 260, 570, 4,530, 8,300

2.2 Answers: Prime and Composite Numbers

YT#1. composite, divisible by 3  YT#2. composite, divisible by 5  YT#3 prime
2. prime  3. composite, divisible by 3  4. prime  5. composite, divisible by 3
6. composite, divisible by 2 (and 3)  7. prime  8. composite, divisible by 7
9. composite, divisible by 3  10. composite, divisible by 5  11. composite, divisible by 3
12. 9, 18, 27, 36, 45, 54, 63, 72, 81, 90  13. 25, 50, 75, 100, 125, 150, 175, 200, 225, 250
14. 11, 22, 33, 44, 55, 66, 77, 88, 99, 110

2.3 Answers: Factors and Prime Factorization

YT#1. 2 2 3 5 or $2^2 \cdot 3 \cdot 5$  YT#2. 1, 2, 3, 4, 6, 8, 12, 16, 24, 48
YT#3. 1, 2, 4, 5, 8, 10, 16, 20, 40, 80

1. $2^2 \cdot 3^2$  2. $2^3 \cdot 3$  3. $2^3 \cdot 3^3 \cdot 5$  4. $2^4 \cdot 3$
5. $2^2 \cdot 5^2$  6. $5 \cdot 13$  7. 13, prime number  8. $5 \cdot 11$
9. $2^4 \cdot 3 \cdot 5$  10. $2^4 \cdot 3^2$  11. $3 \cdot 7^2$  12. $5^2 \cdot 7$
13. $3 \cdot 5 \cdot 13$  14. $2 \cdot 7 \cdot 11$  15. $5^3 \cdot 7$  16. $2^3 \cdot 3^2 \cdot 7$
17. 1, 2, 3, 4, 6, 9, 12, 18, 36  18. 1, 2, 3, 4, 6, 8, 12, 24
19. 1, 3, 5, 15  19. 1, 2, 3, 4, 6, 8, 12, 16, 24, 48
21. 1, 2, 4, 5, 10, 20, 25, 50, 100  22. 1, 5, 13, 65
23. 1, 13  24. 1, 5, 11, 55
25. 1, 2, 4, 8, 16, 32, 64  26. 1, 2, 3, 6
27. 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90  28. 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
29. 1, 2, 3, 5, 6, 10, 15, 30  30. 1, 2, 4, 7, 8, 14, 28, 56
### 2.4 Answers: The GCF (Greatest Common Factor)

<table>
<thead>
<tr>
<th>Y#1</th>
<th>Y#2</th>
<th>Y#3</th>
<th>Y#4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 6</td>
<td>2. 5</td>
<td>3. 2</td>
<td>4. 10</td>
</tr>
<tr>
<td>6. 12</td>
<td>7. 13</td>
<td>8. 11</td>
<td>9. 25</td>
</tr>
<tr>
<td>11. 23</td>
<td>12. 19</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 2.5 Answers: Least Common Multiple

<table>
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<tr>
<th>Y#1</th>
<th>Y#2</th>
<th>Y#3</th>
<th>Y#4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 45</td>
<td>2. 216</td>
<td>3. 315</td>
<td>4. 2280</td>
</tr>
<tr>
<td>6. 40</td>
<td>7. 80</td>
<td>8. 120</td>
<td>9. 525</td>
</tr>
<tr>
<td>11. 72</td>
<td>12. 1260</td>
<td>13. 315</td>
<td>14. 240</td>
</tr>
<tr>
<td>16. 60</td>
<td>17. 120</td>
<td>18. 216</td>
<td>19. 165</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>20. 72</td>
</tr>
</tbody>
</table>